CASIO TEACHING MATERIALS

INVESTIGATING MATHEMATICS

with ClassWiz fx-991CW

Barry Kissane

CASIO

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Contents

Introduction

There are many misconceptions about the place of calculators in school education, the most common of which is that the main purpose of the calculator is to undertake calculations efficiently. While such a function is at times useful, these investigations are offered to provide examples of a more important educational purpose – the learning of mathematics. This set of mathematical investigations is intended to illustrate some of the many possible ways in which teachers and students might use *ClassWiz fx-991CW* productively for teaching and learning purposes.

While calculators are important tools for obtaining numerical answers to questions of interest, their significance for learning will not be fully realised if the only way of thinking about them is as an answering device to be used when all the mathematical thinking has been completed, or even as an answering device in an examination, often characterised by speed and accuracy of response. In this booklet, other ways of thinking about calculators are emphasised. Students might experiment with their calculator in order to see mathematical ideas in a fresh way. They might think of the device as more like a personal laboratory than as a tool. The teacher might contemplate introducing new ideas through students investigating situations with their calculator, rather than turning to the calculator as a helpful device only after all the learning has taken place. Students might even regard their calculator as a responsive toy to be played with to help them to think about mathematical ideas in a new way. For these reasons, the naming of a device is important, in part to break the mindset that its main, or even sole, purpose is to 'calculate' something. The name *ClassWiz* serves this purpose well and it is hoped that the examples of investigations in this booklet capture the spirit of teachers teaching and students learning mathematics with the aid of *ClassWiz*.

An important element of the rationale for this booklet is that, if well-used, *ClassWiz* provides students with mathematical experiences likely to help them understand new ideas. It is powerfully built to do this because of the many kinds of mathematical functionality made available to users – across a wide spectrum of the school curriculum – and its interactive and responsive nature. When a student enters mathematical objects into *ClassWiz*, whether they be numbers, functions, data, measurements or other kinds of objects, *ClassWiz* responds immediately with information that will help them to think about the context in a personal way.

The educational thinking behind this approach was elaborated by Kissane & Kemp (2014), who described a model suggesting that the key contributions of calculators to the learning of mathematics were representation, computation, exploration and affirmation. In brief, *ClassWiz* can *represent* many mathematics concepts and relationships in an interactive way, supporting their learning. Students can learn by undertaking productive *exploration* of mathematical ideas, illustrated throughout this publication using *ClassWiz* in many contexts. When students are encouraged to predict what will happen on their *ClassWiz*, they can see the extent to which their predictions are correct – a process of *affirmation* – or are contradicted; both of these can be expected to support their learning. A more extended analysis of these ideas is provided in the paper itself, available for download from https://atcm.mathandtech.org/EP2014/full/3672014_30001.pdf

Kissane, Barry & Kemp, Marian (2014) A model for the educational role of calculators. In W.-C. Yang, M. Majewski, T. deAlwis & W. Wahyudi (eds.) *Proceedings of the 19th Asian Technology Conference in Mathematics: Innovation and Technology for Mathematics Education*. Yogyakarta, Indonesia, pp 211-220.

We hope that you find these investigations a source of stimulation for both students and their teachers and provide fresh ways of using an excellent mathematical device, *ClassWiz fx-991CW*.

The investigations in this booklet have a number of common features:

- Each investigation occupies a single page, so that they might be easily copied and used in a class with students.
- The investigations are written on the assumption that each student has unrestricted access to a *ClassWiz fx-991CW* for personal use. Some of the investigations might also be used by students with other recent *ClassWiz* models, although it is wise for the teacher to first check whether all necessary calculator operations for that investigation are supported.
- Less ideally, two students might also share a *ClassWiz* between them, although care is needed that students take turns using it and that one student does not dominate use, as well as ensuring that both students are studying the results, not just the student using the device. An advantage of sharing is that it increases the likelihood that students will work together, which is generally beneficial for both.
- The purpose of the investigations is generally to support the development of student understanding of mathematical ideas, so it is important for sufficient time to be allocated for students to engage with the tasks, discuss their understanding with a partner and explore the ideas a little more for themselves. When ideas are new to students, it is important for them to have sufficient time to think about them.
- If students are used to thinking of a worksheet as an invitation to quickly complete some set tasks and routine exercises, and then to check 'the answer', they may need help in realising that a different approach is needed here. Depth of thinking is more important than speed of thinking for most of these investigations. Unlike many worksheets, the main purpose of the investigations is not the development of mathematical skills, to high levels of speed and accuracy.
- Many (but not all) of the investigations are intended to help with the introduction of a new idea, or with deepening understanding of a familiar idea. It is expected that different teachers might use them in different ways with different classes.
- They are generally a little more 'open-ended' than typical school tasks, in the sense that students are invited to undertake some investigations of their own choosing, and it is expected that students will sometimes use the tasks as a launching pad for further investigations. Students are given some responsibility to choose tasks by themselves, rather than merely completing tasks set by the teacher or the textbook.
- They are intended to illustrate student-centred learning, at least to some extent. The focus is on the students' own activity, and the investigations explicitly ask students to think about, talk about and often to write about what they have observed with *ClassWiz*, in order to increase their understanding of some mathematical ideas.
- The investigations are written on the assumption that students are not working by themselves, but have at least one partner undertaking the investigation at the same time. So there are often explicit requests to engage with other people, so that learning mathematics is not a solitary activity and discussion is recognised as educationally valuable. (One way to encourage such joint activity is to provide a copy of the investigation to each *pair* of students, rather than to each individual student, requiring them to work collaboratively to some extent.)
- In most cases, a whole-class discussion after an investigation has been attempted by students will also be productive. If available, a *ClassWiz* emulator (available online) will be useful to support the discussions.
- Each investigation has more than one numbered activity, usually to highlight a different aspect of the mathematics involved. In general, it is assumed that students will undertake these activities in the sequence given.

- Investigations are written to be generally self-sufficient, so that students will not need extensive knowledge of how to use *ClassWiz* before undertaking the task; in some cases, advice is provided in the investigation itself. Teachers will need to decide for their own class whether advice on how to access particular keyboard or menu features is needed, however. The easiest way for teachers to decide this is perhaps to undertake the investigations themselves before presenting them to students.
- While the investigations are presented in the form of printed materials, teachers might at times choose instead to initiate student activities in a different way, using the materials as a prompt. For example, they might choose to demonstrate an idea using a classroom emulator and encourage students to undertake explorations, or they might choose to project an investigation task on a screen, or they might choose to assign an investigation task for homework, rather than providing students with their own copies. Decisions of these kinds are best made by the teacher herself, to suit her preferred ways of teaching and her knowledge of a particular class.
- These investigations have not been developed with a particular national school curriculum in mind, although most are relevant to most modern school curricula. In general, the investigations are sequenced roughly by age and thus by mathematical sophistication, so that investigations appearing earlier are intended for students in the early secondary or middle school years, while those appearing later are likely to be most relevant for senior secondary school students. Of course, individual national curricula will differ slightly in their placement of key ideas, and teachers may choose to introduce new ideas at times of their own choosing.

The following notes accompanying each particular investigation highlight key features and offer some comments about the operation of *ClassWiz* as well as advice on classroom use. They are not intended to constrain teachers to use the investigations in only one way, but rather to encourage them to use them in ways that are productive for their own students, and to adapt them where necessary.

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Notes for Teachers

1. Investigating numbers 1

This investigation is concerned with the relationships between fractions and decimals, and makes extensive use of both the natural display features of *ClassWiz* and the [FORMAT] key for converting between standard and decimal formats. It is designed for younger students, still learning about the relationships between fractions and decimals.

It is important that students have their *ClassWiz* set to use Math mode, through the use of the [≡] menu. Choose *Calc Settings* and then check that the *Input/Output* setting is *MathI/MathO*. This will ensure that both fractions and decimals are displayed as simplified fractions in the conventional way. You will notice a small symbol on the top of the screen to indicate the appropriate setting. This investigation assumes *ClassWiz* is in *Calculate* mode. Use the *Home* key [⌂] to select this if necessary.

The main intention is for students to appreciate that fractions are single numbers (not pairs of numbers), and that many different fractions have the same numerical value, which allows them to be placed at a distinct point on the real number line. The language of equivalent fractions is not used, although students may already be familiar with it. It is a valuable exercise for students to actually plot the fractions on a number line on paper.

Students may at first need some help in entering fractional numbers into their calculator, but this will be quickly mastered (in Math mode … it is sometimes a little more difficult in other modes). The easiest way is to use the [□/□] key. Notice that a fraction like $\frac{3}{10}$ can be entered by first tapping [□/□] and then editing the numerator and denominator separately, or it can be entered more readily with ③ [□/□] ① ⓪.

ClassWiz automatically simplifies fractions to have a denominator as small as possible, so that $\frac{4}{10}$ is represented as $\frac{2}{5}$, but both are the same number, since each is a fractional representation of 0.4.

Make sure that students use the writing space to record their observations, both to help them to verbalise what they have concluded, but also to serve as a basis for discussion with a partner or with the whole class.

Activity 1 introduces the idea that a number might have different representations and the [FORMAT] key allows these to be seen. The screenshots on the page show only decimals whose fractional representation uses tenths, but students will quickly find other numbers for themselves for which this is not the case.

Activity 2 highlights the idea of equivalent fractions, encouraging students to see that any fraction can be represented in many ways. While students are sometimes taught ‘cancelling’ and ‘simplifying’ fractions, often without appreciating the underlying mathematics, the main point here is to see the underlying mathematical idea of equivalence. The third screen introduces the important idea of representing a division as a fraction, which some students might prefer to using the [□/□] key.

Activity 3 draws attention to equivalent fractions, and suggests that students look for further examples of ‘unexpected’ representations of fractions. Encourage students to work together and predict a result before tapping [EXE]. Whole class conversations about equivalent fractions will help students to understand the ideas well here.

2. Investigating numbers 2

This investigation builds on the previous one and is concerned with further relationships between fractions and decimals, focusing on proper and improper fractions and percentages, still making extensive use of both the natural display features of *ClassWiz* and the [FORMAT] key for converting between standard and decimal formats. It is designed for younger students, still learning about the relationships between fractions and decimals and percentages.

Activity 1 is concerned with improper and mixed fractions, and assumes that the *ClassWiz* setting uses the default for *Fraction Result*, which is to represent fractions as improper fractions. It is recommended that this default not be changed – in which case students will need to choose different representations using the [FORMAT] key. Note that the entry of mixed fractions with [SHIFT] [fraction] is not included here but could easily be included if desired, to extend the investigation.

If students are sure that they want a decimal result, a more efficient way than using the [FORMAT] key is to use [SHIFT] [EXE] instead of [EXE]. (Notice on the keyboard that this is shown with the ≈ symbol, recognising that decimals often provide approximate numbers, unlike fractions which are exact.) However, it is not recommended to use this routinely, as it will discourage students from learning about fractions and irrational numbers.

Activity 2 introduces percentages, exploring their representations as fractions and as decimals. The percentage symbol is accessible via the [CATALOG] menu, as described; since students will be entering several percentages in this activity, they might find it convenient to edit the previous command starting with [◀] after the first time, rather than accessing the symbol afresh each time.

Activity 3 explores the use of sexagesimal measure, which (conveniently) can also be used to explore times in hours, minutes and seconds with *ClassWiz*. Notice that numbers represented as fractions can also be shown as sexagesimal numbers using the [FORMAT] menu, although the activity screens do not show this.

3. Investigating variables

This investigation focuses on the use of variables to represent numbers. On *ClassWiz*, the nine available variables are accessed via the [VARIABLE] key. Each variable has only one value at any given moment. A variable can be accessed also via the keyboard, using [SHIFT] with the number keys, while variable x can also be accessed with its own key [x]. The investigation is intended for younger students, just learning about the idea of a variable, but might be useful also for older students accessing variables for various purposes.

Activity 1 helps students become familiar with storing and recalling values of variables.

Activity 2 addresses common misconceptions among young students who don't yet understand the universal symbolic convention that adjacent variables are to be multiplied. It might be helpful for students to see for themselves that AB and AxB give the same result by entering each into *ClassWiz*. Encourage students to change the values of variables to see the effects.

Activity 3 requires students to solve equations, probably informally at this stage. The first screen requires $B = 3$, but there are many (infinite) answers to the other two screens. E.g., for the second screen, $A = 5$, $C = 3$; $A = -1$, $C = 7$; $A = 3.5$, $C = 4$; etc. For the third screen, $D = 1$, $E = 7$; $D = -1$, $E = 7$; $D = \sqrt{7}$, E = 1; etc. Encourage students to look for efficient ways to find several solutions.

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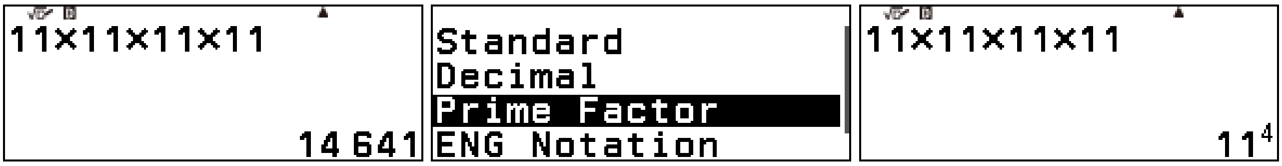
Activity 4 addresses a common misconception with young students, still struggling with the multiplication property of addition, who think that $x(x + 1) = x^2 + 1$. There is only one value of $x = 1$ for which this is true, as students should readily find. Encourage students to use *ClassWiz* to find more examples of this kind.

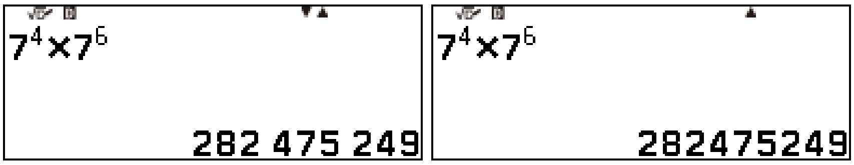
4. Investigating powers 1

The index law $x^a \times x^b = x^{a+b}$ is the key feature of this investigation, and it is assumed that students are not yet familiar with this. The FORMAT menu of *ClassWiz* is critical to resolve numbers into prime factors and thus to reveal some properties of indices. Make sure that students know that the *Prime Factor* command is used only *after* a result is obtained.

If students are less familiar with powers of numbers, it might be useful for them – before starting the investigation – to enter an power of a prime number manually, as in the first screen below, and to then use the FORMAT menu to select the *Prime Factor* command to convert the result to a power. This will help to clarify making clear the meaning of both the base (11) and the index (4), and rehearse a key process used throughout this and the next investigation:



Notice that the *Digit Separator* has been turned on in the settings, as shown in the first screen below to separate thousands with a space. If it is not turned on, large integers are much harder to read, as shown in the second screen below:

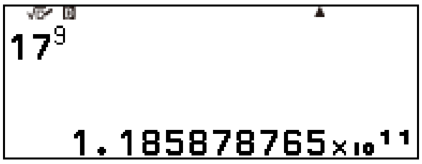


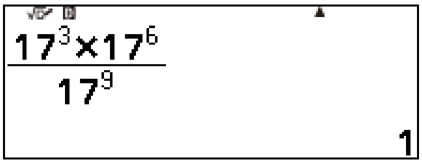
Students can undertake this investigation before getting detailed advice on using the power key or the *Prime Factor* command, but some students might need a little help with these calculator operations in that case. Remind students where necessary to move the cursor out of the exponent of a power (using ▷) before continuing an expression.

It is important that students have their *ClassWiz* set to use Math mode, through the use of the settings menu, so that powers are represented appropriately. Choose *Input/Output* setting to be *MathI/MathO*.

The examples shown in both Activity 1 and Activity 2 mostly use prime number bases, but students will quickly encounter 'strange' results if they use a composite number, following the request to try some more examples for themselves. This will provide students with an opportunity to think about what is happening, prior to it being explored in the following investigation.

The third example in Activity two will generate 17^9, which is too large for *ClassWiz* to represent as an integer, so that scientific notation will be used, as shown below; in that case the *Prime Factor* command will not be available in the FORMAT menu. Students will need to find a different way to check their thinking (most likely through evaluating 17^9 directly and checking that it is the same as $17^3 \times 17^6$). An alternative might be to use a fraction, as shown below:





The second example of Activity 2 deliberately uses powers of a composite number, in case it hasn't occurred naturally with students choosing their own examples. Such examples give rise to opportunities to learn more about how the index laws work, revealing that $(ab)^n = a^n b^n$ and that $a^m \div a^n = a^{m-n}$, which are the focus of the following investigation.

Make sure that students discuss their work with their partner and write down their conclusions, as this will provide a good basis for a whole class discussion about the index law concerned, including perhaps informal proofs of the laws, in addition to being a good introduction to the further work on index laws in the following investigation.

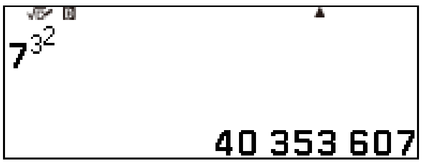
5. Investigating powers 2

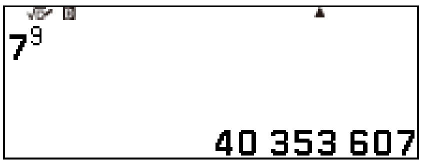
This investigation is an extension of the previous one, which should be completed first. It might be used just before students study exponential functions.

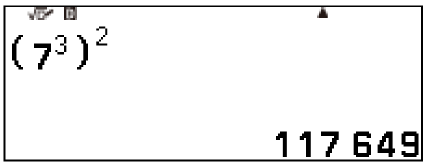
Activity 1 requires some thinking beyond the situations explored in the previous investigation, helping students to see how other index laws can be understood, including those which do not rely on prime numbers, such as $(ab)^c = a^c b^c$. Encourage students to generate and test further examples for themselves and to discuss their observations, some of which might have occurred with the previous investigation. A whole class discussion will be valuable.

Activity 2 and 3 illustrate the index law $(a^b)^c = a^{bc}$. Notice that Activity 3 extends the context to composite numbers.

As an extension, students might also be encouraged to investigate the interpretation by the calculator of repeated use of the exponent key, ▪︎, when parentheses are *not* used:







As shown above, they can see for themselves that the result is interpreted as 7^9 (since $3^2 = 9$), which will make it clear why parentheses were used on the printed sheet to show $(7^3)^2$, also shown above to give a different result.

Make sure that students use the writing space to record their observations, both to help them to verbalise what they have concluded, but also to serve as a basis for discussion with a partner or with the whole class. Encourage students to develop proofs of the various index laws, once they have understood the patterns involved.

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6. Investigating linear functions

The focus of this investigation is the patterns characteristic of linear functions, intended for students not yet familiar with them. These patterns are readily discernible from a table (and of course are visually illustrated well in the case of a graph). Although this is the first investigation using tables, it assumes that students are already familiar with the use of Table mode on *ClassWiz*, including the use of x for the independent variable and the associated commands in the *Tools* ⓞ menu. Help students to use these features of *ClassWiz* where necessary

Make sure that students use the writing spaces to record their observations, both to help them to verbalise what they have concluded, but also to serve as a basis for discussion with a partner or with the whole class.

Activities 1 and 2 involve linear functions, for which students should be able to see the pattern of steady increases (or decreases) between values of the function, which illuminate the idea of the slope of the function. Activity 1 does not involve students using a systematic set of values; encourage them to be systematic when choosing their own x-values to evaluate. Activity 2 will provide a clearer picture of the linear patterns, using the *Table Range* command in *Tools* ⓞ. (Make sure that students understand that *Table Range* describes the *domain* of the functions, not the *range*, however.)

After they have made a prediction of the next number in a table, encourage students to use the ⓥ key to check their prediction with the next row of a table. They can also tap ⓥ to generate the next screenful of (four) rows, which will allow them to test further their predictions.

Notice that the two examples in the third screen of Activity 3 involve a Step of 2 (rather than a Step of 1), requiring students to consider carefully what is happening. The functions in Activity 3 are respectively $f(x) = 3x + 5$, $f(x) = 51 – 4x$, $f(x) = 4x + 11$, $g(x) = 9x + 2$. Encourage students to experiment by entering and exploring their own functions after they have determined which functions are shown in the screens.

ClassWiz makes it easy to generate points to plot on a graph. So, although it is not explicitly referred to in the investigation, it is a good idea to provide students with some grid paper during this investigation and to suggest that they draw graphs of functions, especially if they have not done so extensively before. This will highlight the linear nature of the functions still further, as points will lie on a line.

7. Investigating quadratic functions 1

The quadratic functions in Activity 1 have been chosen to contrast with the linear functions in the previous investigation, highlighting that they do not have consistent slopes. So this will be a good introduction to some of the properties of quadratic functions. Once again, it may be fruitful to provide students with grid paper to sketch the functions concerned.

Students will recognise the first function as $f(x) = x^2 + 1$, but are likely to find it harder to identify the functions in the second screen, $f(x) = x^2 + 4$ and $g(x) = 51 – x^2$. The function in the third screen is $f(x) = x(x + 2)$. Encourage them to persist rather than telling them the answer. Some students might even identify the third function as $f(x) = x^2 + 2x$, providing an opportunity for a class discussion to realise that $x(x + 2)$ is equivalent to $x^2 + 2x$.

Activity 2 addresses this idea of different versions of quadratic functions explicitly. Encourage students to change the Step in the Table settings to see that the two functions are *always* equivalent. While students are likely to be developing the algebraic skills involved to check equivalence, such as expanding and factorising expressions, this kind of experience offers a different perspective on the meaning of relationships like $x(x + 1) = x^2 + x$.

Activity 3 focuses on the shapes of quadratic functions, many aspects of which can be visualised from a table of values. Make sure students have grid paper available to sketch graphs, with the help of the tables of values. Encourage them to visualise the graph before sketching it. The following investigation explores issues of shapes and properties of graphs more directly.

If desired, similar investigations to these could be undertaken at a later stage with other families of functions, such as exponential functions, logarithmic functions and reciprocal functions.

8. Investigating quadratic functions 2

This investigation focuses on a single quadratic function, encouraging students to investigate its graph through the use of *ClassWiz*. Both *Equation* and *Inequality* modes are used. Students might need a brief instruction on how to activate these modes via the *Home* screen with the ⌂ key.

Use of graph paper or grid paper is essential for this investigation; if this is not available, students should construct and label suitable axes for themselves on paper.

Activity 1 uses the quadratic equation $f(x) = 0$ associated with the quadratic function $f(x)$. Make sure that students can extract the necessary coefficients, a, b and c. Encourage students to refer to the function and the equation as separate and important ideas; e.g., they will *graph* the function but *solve* the equation. (Discourage them from using problematic descriptions such as *graphing an equation*.)

Plotting the roots $x = 1$ and $x = -3$ and turning point (-1,-4) will provide a good start to the graph, and help them to appreciate that the line of symmetry is given by $x = -1$.

Activity 2 will help students see the shape of the graph, and the connections between the equation and the inequality. They should be able to visualise the graph after this activity.

Students might be helped by tabulating the function (in *Table* mode) in order to get some further points to make a more careful graph, depending on the precision sought and the level of student sophistication and experience.

After the investigation, students should attempt a similar investigation of the graphs of some other quadratic function by themselves, supported by *ClassWiz.*

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9. Investigating trigonometry

Scientific calculators are often used for trigonometric calculations, and, indeed the presence of appropriate function keys is often a clear symbol that a calculator is of significance beyond arithmetic. In contrast, this investigation provides some examples of how learning about trigonometry might be enhanced by using *ClassWiz* in other ways. It requires students to use *Table* mode, assumed to be already familiar to students, and to have their *ClassWiz* set to degrees, which can be done in *Settings* ☰.

Activity 1 focuses on the relationship for complementary angles that sin x = cos (90° – x), through identifying the appearance of the same numbers in tables for sine and cosine. It is expected that the investigation will also reinforce for students how the ratios of sine and cosine vary over the first quadrant, with sine increasing from 0 to 1, while cosine decreases from 1 to 0. It should be clear to students that these increases and decreases are non-linear.

Make sure that students use the writing space to record their observations, both to help them to verbalise what they have concluded, but also to serve as a basis for discussion with a partner or with the whole class.

Some students might consider checking their relationship by tabulating at the same time both sin x and cos (90° – x), or cos x and sin (90° – x), which will display the identities in a convincing way.

This idea might be extended to supplementary angles, if desired (although note that a Table Step of 5° will generate more table values than *ClassWiz* permits).

Activity 2 is again illustrative of a range of possibilities, focusing on the idea of an identity: that two quantities are the same, regardless of the value of the variable. A table of values shows this kind of relationship very well, and can be further used by changing one of the x-values in the table. This mechanism is a supplement to the idea of a formal symbolic proof of the identity.

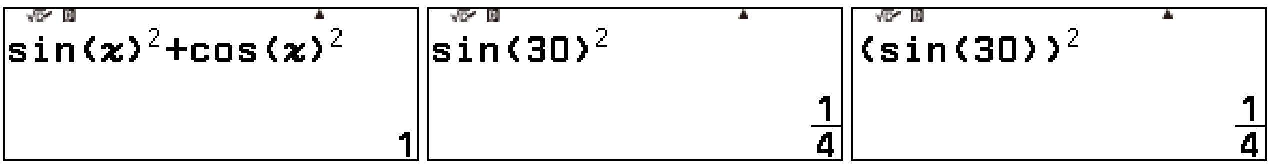
As a further example, the relationship in Activity 1 can also be studied in this way as an identity, with $f(x)$ = sin x and $g(x)$ = cos (90° – x).

If a graph facility is available, graphing both 'sides' of an identity is instructive, too.

The first three further identities suggested are in fact identities, but students should be able to see that the fourth is not an identity. [The correct identity is cos (90° + x) = - sin x.]

At a later stage, more sophisticated students can undertake investigations of these kinds using radians rather than degrees.

Remind students that some conventional (but problematic) mathematical notation like $\sin^2 x$ must be entered into *ClassWiz* carefully, in this case as effectively $(\sin x)^2$, as shown below, in order to avoid a Syntax Error.



The second and third screens above clarify the terminology; notice that these screens show that it is more complicated - and is unnecessary – to use an expression like (sin (30))^2 to represent an expression in conventional mathematical notation such as $\sin^2 30$.

10. Investigating powers 3

This investigation is concerned with powers of numbers and their (eventual) connections with the idea of logarithms, with the fundamental relationship:

$$\log_b(a) = x \text{ when } b^x = a.$$

It is thus concerned with the use of non-integral powers of numbers, which are probably less intuitively understandable to students; through the investigation, students will see that *ClassWiz* tolerates any real numbers as powers, not only integers, in contrast to the two earlier investigations concerned with powers.

It is assumed that students have not yet met the idea of logarithms, which are not mentioned at all explicitly in the investigation. Instead, the investigation is concerned with students addressing the problem of finding good approximations of the power (x) to which a number (b) must be raised to result in a particular result (a), as an intuitive introduction to the concept of a logarithm.

Two different methods of finding the power are explored in Activities 1 and 2 respectively: an iterative guess-and-check process and a process of zooming in on a table of values of an exponential function. In each case, students will be able to find reasonable approximations to the powers efficiently, although a little tediously, and can see directly as part of the investigation that these are sufficiently accurate. To six decimal places, $x = 0.845098$.

Activity 3 involves similar processes (with students choosing their own preference) to find $x \approx 2.089905$ and $x \approx 1.771244$ respectively. Apart from further experience of the thinking involved, students will see that the basic idea is not restricted to 10, but is relevant to other integers as well.

Activity 4 uses a non-integral base (the number ***e***) for the same purpose, to find $x \approx 1.945910$. Students are unlikely to be familiar with ***e*** at this stage, and it is assumed that appropriate development will occur later in their studies. The activity serves to illustrate that the basic ideas underpinning logarithms are not restrictive to integer bases.

Teachers might choose to reveal at a late stage (but only after this investigation has concluded … perhaps the next lesson) that the process of finding the powers they have obtained is made very easy on *ClassWiz*, using the [log▫□] key, and even the (log) ([SHIFT] [▫²]) and (ln) ([SHIFT] [log▫□]) commands on the keyboard, or the three *Func Analysis* commands for logarithms in the catalog ([CATALOG]). These commands can be used to verify the earlier results in the four Activities.

Further studies of the idea of a logarithm might proceed after the introduction provided by this investigation, including with the following investigation.

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11. Investigating logarithms

The first two activities in this investigation are concerned with developing a strong understanding of the idea of a logarithm, intuitively developed in the previous investigation, that it is the power to which a base must be raised to get a particular number. This idea is often emphasised in textbooks; the idea here is for students to experiment personally with it. Once students have a good grasp of this idea, many other aspects of logarithms become more accessible.

It is expected that students will try a variety of bases in each of Activities 1 and 2, so that the fundamental idea of logarithms is not restricted to one particular base (such as base 10, for common logarithms). Students who completed the previous investigation, Investigating powers 3, should include their previous examples, but now using the [log■□] key instead of manually exploring, to see how much effort is saved.

Teachers may at this stage choose to make students aware of the other logarithm commands accessible on *ClassWiz*, including the (log) ([⇧] [■²]) and (ln) ([⇧] [log■□]) commands on the keyboard, and the three *Func Analysis* commands for logarithms in the catalog ([catalog]), as efficient alternatives to the general [log■□] key.

Make sure that students use the writing space for Activity 2 to record their observations, both to help them to verbalise what they have concluded, but also to serve as a basis for discussion with a partner or with the whole class.

Activity 3 is concerned firstly with the helpful property of logarithms that the logarithm of a product is the sum of their respective logarithms. This is the property that made logarithms so useful for computational purposes for hundreds of years. The third screen provides an application of the thinking (using subtraction instead of addition) to appreciate that the result is $\frac{5}{8}$, which is given exactly on *ClassWiz* because of its very high level of precision. Make sure that students commit to their expectation in writing before affirming it with *ClassWiz*.

Notice that the idea of an 'antilogarithm', for which tables have been provided in the past, is replaced on the calculator by a process of raising the base to a power. *ClassWiz* permits the use of the exponentiation key [■□] for this purpose. If one is available, teachers might choose to show students a previous table of logarithms and antilogarithms – or even a slide rule – to vividly demonstrate how much computational efficiency has been achieved through the use of calculators.

Activity 4 also invites students to explore other logarithmic properties (such as the logarithm of a power); they might also explore the logarithm of a quotient or of a root, too. More sophisticated students can undertake investigations of this kind using natural logarithms or logarithms to other bases to explore the generality of the concepts.

12. Investigating univariate data

This investigation is concerned with understanding the use of univariate statistics to summarise a set of data. It assumes that students have already learned how to use the Statistics mode of their *ClassWiz* with univariate data, and thus is appropriate for use after an introduction to the topic.

Activity 1 is concerned with transforming a data set additively, a form of translation, which has the same effect on the mean and the five-number summary data (minimum, Q_1, median, Q_3 and maximum) , but which has *no* effect on measures of spread. (To enter the adjusted data, students should be able to start with the initial data given, and then edit each value in turn, replacing it by the next higher value). Students should be able to readily compare their results with the printed results on the Investigation sheet.

Activity 2 is concerned with a multiplicative transformation, in which each data point is multiplied (by 100 in the first instance). This scaling will affect both the mean and the standard deviation in the same way (multiplying each by the same factor), but will affect the variance by the *square* of the multiplier. (This is one of the reasons that the standard deviation is often preferred as a measure of spread over the variance – it is scaled in the same way as the original data.) Again, students should be able to readily compare their results with the printed results on the Investigation sheet.

Make sure that students use the writing spaces to record their observations, both to help them to verbalise what they have concluded, but also to serve as a basis for discussion with a partner or with the whole class.

Activity 3 invites students to consider a different multiplicative transformation. A later investigation might examine the effects of a transformation that involves *both* a translation and a scaling – a good example is a set of temperatures in the Celsius scale that is converted to a set of temperatures in the Fahrenheit scale by the transformation $F = 1.8C + 32$.

13. Investigating arithmetic sequences

This investigation allows students to explore arithmetic sequences by actually constructing the sequences efficiently on *ClassWiz*; school curricula generally develop formulas for various purposes, generalising the observations, but this investigation works instead directly with the sequences themselves. Both approaches of course are important to give students a solid understanding of the concept of an arithmetic sequence.

Activity 1 uses the design of the calculator (and many other calculators) to generate terms of a sequence by repeating the same operation, in this case addition of a constant number (the common difference). Once the operation of adding is entered, successive terms of the sequence are easily generated by tapping the EXE key in succession. Notice that students do not use the Ans key – *ClassWiz* adds the expression using *Ans* automatically. The tenth term of the sequence is easily found (with careful counting of steps) to be 45.

Activity 2 uses the same idea, although with a negative common difference, so that terms become successively smaller. This permits many questions about sequences, like those posed, to be directly answered. With a common difference of -6, the 5th term is 33 and the first ten terms are positive.

Activity 3 overcomes the obvious limitation of the previous two activities – that when many terms of a sequence are required, many successive taps of EXE are needed, which can be both tedious and difficult to count. Generating a sequence via a function allows a table of values to be easily constructed and used; the advantage over the previous mechanism is that the terms of the sequence can be scrolled easily. Students might need some help to construct suitable formulas, but the problem is important to solve as it makes clear the nature of the sequence, with a fixed starting point and successive additions of the common difference. It seems better to leave the formula in a form like $f(x) = 9 + 4(x – 1)$ instead of a 'simplified' form of $f(x) = 4x + 5$, to preserve its meaning.

While the table also has limitations – because *ClassWiz* limits the number of rows displayed in tables – the formula used to generate the terms will allow students to find any term of the sequence, using the function with f(x) in Calculate mode, allowing students to find that the 100^{th} term is 405.

Activity 4 allows students to explore the same sequence as in Activity 2, using $f(x) = 57 – 6(x – 1)$. Again, it is unnecessary (and would be unwise) to 'simplify' this to $f(x) = 63 – 6x$.

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14. Investigating geometric sequences

As for the previous investigation, this investigation allows students to explore geometric sequences in the same way: by actually constructing the sequences efficiently on *ClassWiz*; school curricula generally develop to find formulas for various purposes, generalising the observations, but this investigation works instead directly with the sequences. Both approaches of course are important to give students a solid understanding of the concept of a geometric sequence.

Activity 1 uses the design of the calculator (and many other calculators) to generate terms of a sequence by repeating the same operation, in this case multiplication by a constant number (the common ratio). Once the operation of multiplication is entered, successive terms of the sequence are easily generated by tapping the (EXE) key in succession.

Make sure that *Digit Separator* is turned on in settings (☰), so that large numbers are more easily read. The 16th term is the first that is large than 50 million. A class discussion about the (rapid) speed of exponential growth in situations like this may be appropriate.

Activity 2 uses the same idea, although with a common ratio less than 1, so that terms become successively smaller. These processes permit many questions about sequences to be directly answered. It also raises issues of the eventual convergence of the sequence (to zero) although the physical model clearly doesn't handle this situation very well. Students might discuss these issues informally in a class discussion.

Activity 3 overcomes the obvious limitation of the previous two activities – that when many terms of a sequence are required, many successive taps of (EXE) are needed, which can be both tedious and difficult to count. Generating a sequence via a function allows a table of values to be easily constructed and used; the advantage over the previous mechanism is that the terms of the sequence can be scrolled easily. Students might need some help to construct a suitable formula, but the problem is important to solve as it makes clear the nature of the sequence, with a fixed starting point and successive multiplications by the common ratio.

While the table also has limitations – because *ClassWiz* limits the number of rows of tables that can be displayed – the formula used to generate the terms will allow students to find any term of the sequence, using the function with (f(x)) in Calculate mode. However, students should be wary of assuming that mathematical models of this kind are appropriate – or accurate – for very long time periods, because of the (unreasonable) assumption about a constant growth rate in the context described. Similarly, students might reasonably question the context of a city having a population of (exactly!) 8 million people. The last question in the activity might prompt some valuable class discussion about such issues. (This issue also arises in the first activity in *Investigation 21: Investigating population models.*)

Note that the formula in Activity 3 used units of a million. It is not incorrect, but is more awkward to use integers instead, so that a formula of $f(x) = 8000000 \times 1.02^{x-1}$ will also correctly show sequence terms. However, on *ClassWiz*, this function is larger than the screen width – and so is difficult to read; in addition, tabulated terms will be shown in scientific notation in the table generated, even though they can be seen in full when scrolling. A discussion of such issues with the class may be valuable, and students encouraged to make their own choices, as well as to see that other choices are valid.

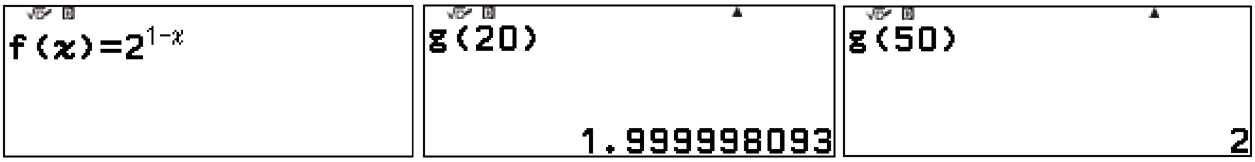
15. Investigating series

This investigation continues the preceding two investigations, exploring the relationships between series and their associated sequences. It is assumed that students have completed the previous two investigations before starting this one.

The key idea on *ClassWiz* is to use a summation function to add successive terms of a sequence, as demonstrated in Activity 1 (which uses the same sequence from *Investigation 13: Investigating Arithmetic sequences*). The expression used, $\sum_{x=1}^{x}(f(x))$, might be slightly problematic for some students, as it uses the same symbol (x) for both the summation counter and its maximum value, and a class discussion of the meaning of the symbol may be helpful. Perhaps the easiest way to interpret the symbol is to examine the successive terms of $g(x)$ in the table, so that students can see how it evaluates successive terms of the series. The 6^{th} term of the series is 114.

Note that examining a new series requires only changing the definition for the sequence. The series function $g(x)$ is always the same, regardless of the sequence function. In this case, changing the defining function to $f(x) = 7 + 5(x - 1)$ will show that the 5^{th} term of the series is 85.

Activity 2 uses the series generated by the sequence $f(x) = 2^{1-x}$ for $x = 1, 2, 3, \ldots$ While a table of values is a natural way to explore this series, terms of the series can be evaluated directly using f(x) in Calculate mode, as shown below:



After 20 jumps, the frog will have gone almost 2 metres, to the edge of the circle. After 50 jumps, the third screen above suggests that it will have gone 2 m. This series converges (although the language of convergence is not likely to be used by students yet) and the results suggest that the frog has reached the edge of the circle after 50 jumps (which is problematic, as it jumps halfway to the edge on each jump). Class discussions about this will help students develop an intuitive understanding of the difficult concept of an infinite limit, explored further in *Investigations 19 and 20: Investigating Limits 1 and 2*). [Remind students that *ClassWiz* reports a result of 2 because that is the best approximation to the actual value (to 23 places of decimals) which is (very) slightly less than 2.]

Activity 3 describes a practical situation requiring a series (of triangular numbers), generated from the sequence of natural numbers, $f(x) = x$. Using a suitable table, students should be able to find that there are 91 cans in the thirteen rows and that 20 rows are needed to get 210 cans.

16. Investigating randomness

This investigation makes use of the two random number functions on the *ClassWiz*, *Ran#* and *RanInt*, both included in the *Probability* catalog and also requires students to be comfortable with the use of tables. The *Ran#* function generates a 3-digit random number uniformly distributed on the interval (0,1), while *RanInt(a,b)* generates a random integer between a and b (inclusively) with a uniform distribution. It is assumed that students already have some familiarity with these functions. The essential point of the investigation is to see in practice how well theoretical ideas related to probability and chance apply in practice, using various kinds of simulated data.

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By the very nature of randomness, results are unpredictable to some extent. Accordingly, in all three activities, it is very important for students to compare their answers with others and it is essential for the results of the whole class to be discussed in some suitable way, as this will increase the extent to which the long-term relationships become clear. To help these discussions, make sure that students use the writing spaces to record their observations, both to help them to verbalise what they have concluded, but also to serve as a basis for discussion with a partner or with the whole class.

Activity 1 focuses on the idea of a uniform distribution, and gives some insight into what this means in practice, with the (pseudo-) random numbers generated by *ClassWiz*. Expect that there will be some anomalies in the short term (such as students with substantially more or less than half the numbers below 0.5), but these can be explained as short-term fluctuations if enough data are generated collectively to see a ‘big picture’. It is important to share student results to an extent to counter the misconception that half of the numbers will be less than 0.5 every time, regardless of the sample size. A useful further strategy is to aggregate class results to see a big picture emerging. Encourage students to conduct other experiments, as well as the two suggested (proportions less than 0.5 or 0.3).

Activity 2 highlights the *ClassWiz* practice of producing numbers rounded to three decimal places, and then representing them to two or one decimal place if trailing digits are zero. Once again, short-term variations will be common, but the compilation of results will help students to see that they are merely short-term fluctuations. Make sure that they commit to a guess before generating any data – the easiest way to do this is to ask them to write down their guess on paper and share it with a partner. Encourage students to repeat the experiment a few times, rather than simply once, to get a feel for the variation involved.

Activity 3 offers a way for students to explore the familiar problem of tossing a pair of coins. Many students (naïvely) think there are three possibilities: two heads, two tails or one of each, with probabilities of $\frac{1}{3}$ for each. The simulation is intended to help them to understand why this is incorrect, as a head followed by a tail (HT) is a different outcome from a tail followed by a head (TH), and so the probability of getting two heads is only $\frac{1}{4}$.

As for the previous two activities, stronger and more convincing results will be obtained by pooling the results of individuals into pairs and of pairs into the whole class to stimulate a discussion. A discussion might include observations that some students have different impressions after a small number of tosses than other students, but that the pooling of results (more data) gives a more reliable impression of what happens in the long run.

An alternative approach to generating each coin toss separately is to generate a sum of the number of heads as a single function: $f(x) = RanInt(0,1)+RanInt(0,1)$. This will result in 0, 1 or 2 (heads), and make it easier to see the totals more quickly, and probably easier to see that a result of one head occurs around half of the time. It is also suggests a method of handling the tossing of more than two coins, should students choose to extend the activity.

17. Investigating coin tosses

This investigation uses the *Coin Toss* app in *Math Box*, which is a flexible facility for simulating tosses of fair coins (i.e., the probability of Heads and Tails is 0.5 for each.), and much easier to use than the methods used in *Investigation 16: Investigating randomness*. This facility allows students to design various experiments and see how the simulated results match the theoretical probabilities in efficient ways.

Activity 1 involves simulating a pair of coins, at first with a small number of replications, which will help students to see that random results can be quite variable. Ideally, they should firstly toss a pair of physical coins and record the results, comparing them with others and perhaps the whole class. Some naïve students might at first think that three outcomes (two heads, two tails, tail and head) are equally likely and expect each to occur about one third of the time. While examining results in a list on *ClassWiz* is inefficient, it will give students a sense of what is happening, with less physical problems of tossing and catching coins. When class results are aggregated, it is likely that the naïve expectation will be seen to be problematic, as about half of the simulated tosses are likely to be a Tail and a Head. A class discussion and a suitable diagram may help students counter the misconception of all outcomes being equally likely and even to develop the theoretical probabilities involved.

Activity 2 provides a more substantial amount of data (250 simulated tosses), as well as a more efficient way of summarising it (using relative frequencies). Draw students attention to the differences between their own results and those on the sheet. (It is almost certain that students will each get a different relative frequency distribution, and some may expect that their result should match what is written.)

Make sure that students understand that the two representations with *List* and *Relative Frequencies* are using the *same* simulated data (perhaps most easily seen with a small sample) – so they can toggle between the two by first using the ⮌ key. If they use ⮌ ⮌ to return to the set up screen, *ClassWiz* will generate a fresh set of data.

Apart from the number of repetitions, Activity 2 is essentially the same context as Activity 1, and offers a glimpse into what happens in the long run. Individual student results are likely to be closer to the theoretical probabilities of ½, ¼, and ¼ than were seen in Activity 1, and students working in pairs (to aggregate their results) or in the whole class (to aggregate all the results) will see more compelling evidence of the theoretical probabilities.

Through these activities, students will also see the inevitable random variation, thus helping to distinguish the theoretical probabilities from the long-term relative frequencies, an important understanding of a key idea.

The two activities in this investigation provide an opportunity for students to see how the app works, and to compare the results with their expectations. Teachers will be able to generate further activities of these kinds, should they wish.

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18. Investigating dice rolls

This investigation is a companion to the previous one, with similar broad intentions of providing stochastic experience. It uses another *Math Box* app called *Dice Rolls* to simulate rolling of fair six-sided dice. It assumes that students have already encountered the previous investigation.

While rolling a pair of dice is well studied in schools, and often features in children's games, Activity 1 investigates rolling three fair dice. This context is not generally studied in school, because it is a little more difficult for students to derive the theoretical probabilities (as there are 216 outcomes). Students should be able to determine the minimum (3) and maximum (18) scores for the three dice.

Both list and relative frequency summaries are suggested. Again, after students are familiar with the operation of the app, encourage them to compare results with others, or even the whole class to acquire a sense of the distribution of likely results.

As with Coin Toss, remind students understand that the two representations with *List* and *Relative Frequencies* are using the *same* simulated data (perhaps most easily seen with a small sample) – so they can toggle between the two by first using the ⓢ key. If they use ⓢ ⓢ to return to the set up screen, *ClassWiz* will generate a fresh set of data.

A class discussion might also help students to appreciate that sometimes theoretical probabilities are difficult to determine, but a good approximation is available with well-organised and large collections of simulated data. Of course, *ClassWiz* experiments like these are relatively modest in size, but will help students understand the significance of such Monte Carlo methods.

Student results (especially if aggregated) will likely be close to the theoretical results, which include Prob (10 or 11) = 25% and Prob (9 or 12) ≈ 23%, so that almost half of the totals are likely to be in the set, {9, 10, 11, 12}. [Some students might be able to compare this with the case for rolling two fair six-sided dice, for which almost half the totals are likely to be in the set {6, 7, 8}.]

Activity 2 has a similar structure to previous investigations, and again explores a context not always routinely addressed in school, concerning the *difference* of two dice. Again, students will obtain a sense of likely results from their simulations, as well as seeing that the results are not completely predictable and that further data provide stronger results. As for Activity 1, some students could derive the theoretical probability distribution of the differences, although it is unusual for these to be treated in school. For reference, here is the theoretical distribution:

Difference	0	1	2	3	4	5
Probability	6/36	10/36	8/36	6/36	4/36	2/36

Class discussions about the consistency of results and (perhaps) comparison with theoretical probability distributions will be valuable for activities of this kind.

As for the previous investigation, the two activities in this investigation provide an opportunity for students to see how the app works, and to compare the results with their expectations. Teachers will be able to generate further activities of these kinds, should they wish.

19. Investigating limits 1

This investigation explores two methods of using *ClassWiz* to understand limiting processes for functions as well as to get good approximations of a limit. These are numerical processes and it is assumed that at some point students will deal more formally and theoretically with limits; the idea here is to give students personal experiences of the processes to complement formal proofs.

Activity 1 is concerned with a function that is undefined at a point, but well-defined elsewhere, and helps students to use a definition of the function in f(x) to study the values of the function as the values of the variable get closer and closer to the point concerned. The idea of 'closer and closer' is regularly involved in discussions about the nature of a limit; this idea is put into practice here in a direct way. Students should not have difficulty seeing that the limiting value is 4, and get a sense that the closer and closer they get to $x = 2$, the closer and closer the value of the function gets to 4.

Activity 2 deals with a well-known trigonometric limit in a different way, using a table of values. Remind students, if necessary, that replacing θ with x in the function and limit definitions does not change the result, but is necessary because *ClassWiz* only accepts the variable x. While the approach here is different from Activity 1, the idea is the same: taking values of x closer and closer to $x = 0$ will reveal that the values of the function get closer and closer to a limit, which in this case is 0.

Limits to infinity cannot be handled in quite the same way, but Activity 3 illustrates some similarities. An approximation of $x \longrightarrow \infty$ can be obtained by using larger and larger values of x. In this case (and in many other cases), the limiting value (in this case -3) becomes clear relatively quickly. In fact, adding one extra 0 to the end of the number shown on the sheet will generate a (rounded) result of -3. *ClassWiz* will tolerate numbers up to 10^{99} as values for x in this context, should students wish to get even closer to a limiting value, while still remaining finite

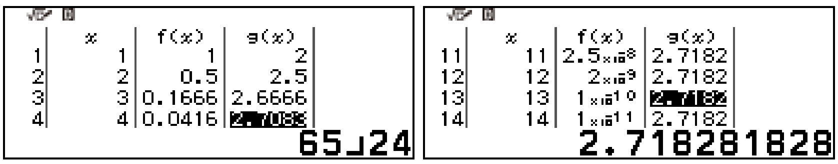
Again, students will – and should – learn more formal procedures for finding limits, which involve mathematical proofs, possibly after investigations like these; it is expected that experiences like those provided in these activities will add some insight of a different kind, however.

20. Investigating limits 2

This investigation extends the processes used in the previous investigation to consider the limits of sequences and series and is intended for older students who have encountered ***e***.

It would be helpful for students to have encountered both sequences and series on *ClassWiz* before undertaking this investigation, especially *Investigation 15: Investigating series*, in which the processes of constructing series from sequences are used.

Activity 1 deals with a famous series, first described by Euler in 1748 (long before *ClassWiz* was invented). Note that the series definition for $g(x)$ begins with $x = 0$, as the first term of the sequence is defined for $x = 0$. The sequence terms approach zero very rapidly while the series converges remarkably quickly to $\boldsymbol{e} \approx 2.71828182845 \ldots$ with the same result given to nine places of decimals after the 13th term, as the screens below show:



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Students at this level should recognise e as it appears and can see an approximate value for it on their *ClassWiz* with ⇧ ⑧.

Activity 2 shows another way of obtaining e as the limit of the sequence:

$$e = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$$

However, students will notice that the limit approaches e very slowly in comparison with the series in Activity 1. This might be a stimulus for a class discussion about rates of convergence, depending on the student sophistication.

Activity 3 explores another famous infinite series – the *Harmonic Series* – as an example of a series that does not have a limit, even though the sequence from which it has been constructed has a limit (of 0, as for the sequence in Activity 1). Note that the series definition for $g(x)$ begins with $x = 1$, as $f(x)$ is undefined for $x = 0$, unlike that for Activity 1. Students will see that the value of the series increases (very slowly) as the number of terms increases, in sharp contrast to the rapid convergence of the series in Activity 1. Because of the slow increase, a table will be less useful than evaluating the series function $g(x)$ directly using (f(x)) in Calculate mode.

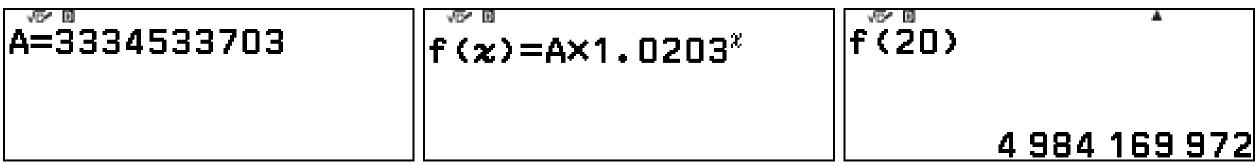
Students can see that the first 100 terms add to about 5.19, the first 1000 terms add to about 7.49, while the first 10 000 terms add to about 9.79, demonstrating the very gradual increase in the series value, in marked contrast to the rapid convergence seen in Activity 1. In addition, *ClassWiz* requires some minutes of processing time to obtain the last result, noticeably much slower than the earlier rapid processes.

21. Investigating population models

This investigation is concerned with modelling population growth in different ways. It is set in the context of the world population, to illustrate the ideas concerned, but it is expected that at some stage students will use the same techniques to study similar questions for their own country. In that case, they may need some help regarding how to locate suitable credible data for their own country. The *Worldometers* site is a good possibility, although others might be preferable for teachers and students.

Activity 1 involves the use of an exponential model, while Activity 2 uses the bivariate statistics capabilities of *ClassWiz*. It is assumed that students already have some familiarity with the bivariate statistics capabilities, so you will need to monitor their work if this is not the case and provide help where appropriate.

The appropriate model in Activity 1 is an exponential model assuming a constant growth rate of 2.03% per annum. Students may need help to think of an increase of 2.03% as equivalent to multiplying by 1.0203. Rather than entering large numbers several times into *ClassWiz*, encourage the students to store them as variables using (⇄x). Then use (f(x)) to define a suitable function, which can be efficiently used to make various predictions, as shown below:



Under an assumption of a constant growth rate, the model in 1965 would have predicted a world population of about 4.98 billion people by 1985, which is reasonably close to the actual figure of 4.87 billion. However, other predictions – extrapolating further – are likely to be increasingly problematic.

To use the model to estimate when the world population will reach 10 billion, a convenient method using *ClassWiz* is to solve an equation using the *Solver* in Equation mode, as shown below:

The solution of about 55 years (after 1965) or the year 2020 was clearly not very satisfactory, as the world population in 2020 (according to *Worldometers*) was in fact only about 7.9 billion people (because the annual growth rates were in fact not a constant 2.03%, as the model required.)

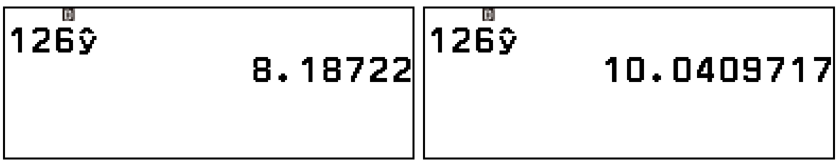
Encourage students to discuss this model, recognising its assumptions and flaws. Encourage them also to be careful to not assume that all digits provided by their model are likely to be valid – of necessity approximations are involved.

Activity 2 adopts a different approach of imagining someone was trying to predict population growth based on available statistical data, which involves selecting suitable models. Thus the data given seem to fit a linear model very well (as the correlation coefficient is $r \approx 0.99$), which is often the case over relatively short periods, even with real data like these.

Population models are often exponential however, as we know that population growth is often approximately exponential in character. So the activity also illustrates how the population for 1985 was predicted to be about 4.80 billion, reasonably close to the actual value of 4.87 billion. Depending on their experience with Statistics mode, students might need help in both changing models and using models to predict.

Encourage students to plot data on graphs in order to help choose a suitable model.

ClassWiz allows students to make predictions from data with a choice of statistical model. In this case students might compare the linear and exponential models to predict the world population in 2026, reported by *Worldometers* to be about 8.3 billion. Since only data up to 2000 have been used, students can see how successful predictions *made in that year* might have been. The two predictions are shown below, with the linear model shown first:



In this case, perhaps surprisingly, the linear model is much better than the exponential model, which overestimates world population growth quite severely (most likely because growth rates dropped significantly in the early years of the 21st century, violating the (implicit) assumption that the rates are constant.

Encourage students to use the writing spaces to record their observations, both to help them to verbalise what they have concluded, but also to serve as a basis for discussion with a partner or with the whole class.

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Make sure that students understand that the models studied here are simplistic to an extent: each assumes that population growth rates are static over time, when clearly this is not the case. Although a mathematical model provides a convenient way of estimating the future and exploring the consequences of hypothetical scenarios; the results must always be interpreted in the light of the assumptions involved as well as the data involved. (Note that data may be reported differently by different sources or even within the same source.) Better models might be expected if more data are used to construct them. More sophisticated models might be available to students at a later stage, but the basic ideas can be explored profitably with these relatively simplistic approaches.

This investigation might support a class discussion of mathematical models used to predict the future, making clear some of the pitfalls possible, especially with the assumptions made (which are not always made clear in public or media discussions). Recent prominent examples include modelling pandemics, climate change and financial indices.

22. Investigating complex numbers

While complex numbers are accessible in Complex mode on *ClassWiz*, they also appear sometimes in Equation mode when solutions of polynomial equations are complex. In this investigation, students will need to use Complex mode. The default setting for complex numbers, rectangular representation, is used; this can be changed in ⊜ if desired.

Activity 1 is concerned with representations of complex numbers, both on *ClassWiz* and on an Argand diagram. Make sure that students plot several complex numbers on paper, to understand the relationships between the two representations. They will need a ruler and a protractor to help with this process, to verify the polar forms of the numbers. Note that if *ClassWiz* is set to use radians instead of degrees in ⊜, students will see arguments in radians (which are less helpful for plotting on the diagram.)

Activity 2 is designed to help students to see how existing *ClassWiz* capabilities can be extended through the use of the de Moivre Theorem, which is likely to be prominent in their early studies of complex numbers. So that students understand the problem being addressed, ask them first to find $\sqrt{8-6i}$ directly from the *ClassWiz* keyboard using (√■), which will give a Mathematical error.

Some students may need help storing a number in the x memory using (STO), which is a very useful skill in this activity, as it avoids excessive keyboard use. [Note that, while complex numbers can be store in memories in Complex mode, they cannot be recalled in other modes, but will result in a Mathematics error.]

Prompt students if necessary to see that the other square root of $8 - 6i$ is $-(3 - i) = i - 3$, which they can also check by squaring.

If necessary, encourage students to reduce keystrokes when finding further square roots by first editing the stored value of x and then tapping (EXE) again while the complicated expression is still on the screen. This is much easier than entering the expression a second time.

They should find that $\sqrt{32+24i} = 6 + 2i$ and $\sqrt{3+4i} = 2 + i$, which can be easily checked by squaring. When they plot the associated pairs of square roots on an Argand diagram, they should notice their symmetry around the origin.

23. Investigating derivatives

ClassWiz provides numerical derivatives, not symbolic derivatives, such as those available through computer algebra systems, such as CASIO's *ClassPad*. This investigation uses this capability to help students understand the nature of derivatives and to use the patterns involved to see some general results. Students studying introductory calculus will of course study formal symbolic methods as well; investigations of this kind are intended to add meaning to those studies, and so are appropriate in early stages. It is helpful for students to think about the numerical derivative informally as a way of describing the rate of change of a function at a point.

Activity 1 introduces students to the *ClassWiz* command to find the derivative at a point, highlighting that a function can be defined first or a derivative can be obtained directly from the function rule (without formally defining the function).

Help students to use their *ClassWiz* efficiently: if a derivative of a function is examined at several points, as suggested in this activity, it is easier to use the ⓘ key after the first instance and to then edit the point concerned, rather than to re-enter the derivative command and its argument each time.

Students will find that the derivative of a linear function is simply its slope, regardless of the point chosen, which ought not be surprising to them, assuming they are already familiar with the idea of the slope of a linear function, representing the rate of change of the function. Encourage them to discuss this with their partner – or more widely – and to write down their conclusions to understand that linear functions (unlike other functions) have a constant derivative. Make sure that they try other linear functions to consolidate this important idea.

Activity 2 invites students to explore derivatives of a non-linear function (in this case a quadratic function). Help students to see, if necessary, that function g finds the derivative of f at *any* point, not just a particular point, shown by the setting $x = x$. The resulting table with both the function and its derivative at many points will show clearly that the derivative is not constant, in marked contrast to the linear case, but that it changes at every point. Some students at this level ought to be able to detect the pattern in the derivatives as $g(x) = 2x – 2$, Changing the *Table Range* settings will reveal that the pattern does not depend on the settings, but seems to be always the case.

In their discussion, and in their writing, students should interpret this relationship as providing good information about the graph of the function, in this case indicating that the graph is curved everywhere and not linear anywhere. For example, when $x > 1$, the derivative is positive, while for $x < 1$, the derivative is negative; and of course when $x = 1$, the derivative is zero, indicating a turning point of the parabola. Encourage students to use what they have found to sketch a graph of f or do that as a whole class discussion.

Activity 3 is similar to Activity 2, except that it is suggested that only the derivative be tabulated, which will help focus attention on the patterns involved, in this case that $g(x) = 2x + 3$.

Students can easily examine other quadratic functions in this way, merely by changing the definition of f, and should be encouraged to do so.

Indeed, other kinds of functions can be treated in the same way – perhaps at a later date – to increase insight into the nature of the derivatives of functions.

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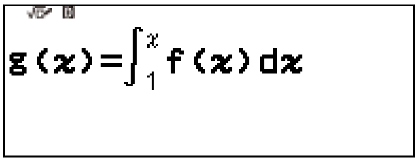
24. Investigating integrals

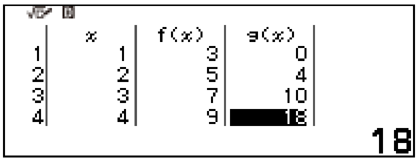
As for derivatives, *ClassWiz* can evaluate definite integrals of elementary functions numerically, not symbolically like computer algebra systems, such as CASIO's *ClassPad*. In this investigation, students use this facility to improve their understanding of the nature and meaning of definite integrals.

Activity 1 serves both to engage students with the available capabilities and to explore their geometric meaning. They should recognise the three shapes involved, especially after comparing their observations with their partners. The first is a rectangle with height 7 and length 11, with an area of 7 x 11 = 77. The second is a trapezium between $x = 2$ and $x = 5$, with one side along the x-axis and the opposite side along the line $y = x + 3$; so the area is $3\left(\frac{5+8}{2}\right) = \frac{39}{2}$. The third is a semicircle of radius 2 with area 2π. Adjusting the limits of integration will be a useful investigation, consolidating students' thinking.

Activity 2 will provide students an opportunity to detect patterns in the definite integrals. Encourage them especially to work together with a partner. Help students to notice that the integral defined by g uses 0 as the lower bound and x as the upper bound, so that each definite integral finds the integral up to *any* point (in a similar way to the derivatives in the previous investigation.) As with other tables, it is always a useful strategy for students to predict the next line of the table and then tap the ⓥ key to affirm their thinking. In this case, the pattern can be seen to be $g(x) = x^2 + x$, which some students will first notice as $g(x) = x\,(x + 1)$.

When students change the lower bound of the integral, as suggested, they should see that the same kind of patterns are involved, but with a constant added or subtracted. Here is an example:





Comparing the tabulated values with those in the Activity will help some students to see that now $g(x) = x^2 + x - 2$. Discussion of this phenomenon will strengthen their understanding of what is happening.

Activity 3 is somewhat more advanced than the previous two activities, addressing the definition of the natural logarithm function, *ln*. Draw students' attention to the lower bound of the integral (which is 1 instead of 0). This activity might be better introduced later in a calculus course than the previous two, when natural logarithms and ***e*** have both been introduced.

25. Investigating transformation matrices

Transformation matrices provide a useful way of describing various transformations in the plane. This investigation explores these to understand how and why they have their effects, using Matrix mode on *ClassWiz*.

Activity 1 provides a matrix **A** and suggests that students experiment to see what effect it has on individual points. Make sure that they understand the need for *pre*-multiplication (although if students use post-multiplication, they will find an error because the dimensions do not match). Students should not have too many difficulties observing that the matrix reflects a point in the x-axis, as x-values are left unchanged and the y-values are multiplied by -1. Suggest that students use grid paper to graph the points and their images to make the relationship clearer if necessary.

Students can use this activity as a motivation to study carefully how the various elements in the matrices are involved in the multiplication, by considering carefully the procedures for matrix multiplication.

Activity 2 extends the idea of transformations to other matrices, for which students might be able to successfully predict their effects, or might need to choose a few examples to provide pointers to the matrix effects. Again, working collaboratively with other students and using grid paper to study some examples will be useful strategies here. The three matrices respectively reflect a point about the y-axis; bring about a vertical stretch by a factor of 3; and reflect in the line $y = x$, followed by a reflection in the x-axis (which together is a rotation of 90 degrees clockwise about the origin).

Activity 3 extends the idea of transforming a point to transforming a set of points (which might define a two-dimensional shape). The activity uses a triangle, comprising three points. Students should be able to observe fairly easily the effects of the transformations and so might imagine how such transformations are used in practice (such as with screen animations). Graphing the triangle and its image will help them to see that the transformation in this case has doubled the x-values and left the y-values the same, resulting in a horizontal stretch by a factor of 2. So the transformation matrix is

$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

Activity 4 encourages students to investigate various transformation matrices of their own choosing to see their effects on the square. Again, grid paper will be helpful to compare their predictions on *ClassWiz* with their drawings.

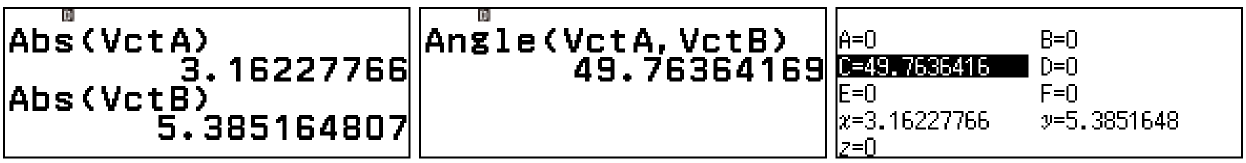
26. Investigating vectors

ClassWiz provides a mechanism for students to define vectors and then investigate various vector operations, using Vector mode.

Activity 1 both engages students in the ways of defining and dealing with vectors, as well as encouraging them to look closely at how vector addition takes place. Students should not have too much trouble seeing the relationship between the vector components of **a**, **b** and **a** + **b**, which will be helped by drawing the vectors on a coordinate plane on paper, demonstrating the Parallelogram Rule. The *ClassWiz* sum will make it clear that components are added to get the sum. Encourage students to try some other examples.

Activity 2 extends the ideas of Activity 1 to include both subtraction of vectors as well as linear combinations of vectors. Again, both the drawings on paper and the analysis of components will give students a good sense of how these operations are defined and executed. Students will benefit from devising similar examples for themselves, and checking that the *ClassWiz* operations and the diagrams match each other.

Activity 3 extends the ClassWiz operations to include the length of vectors (given by their absolute value) and the angle between vectors. Encourage students to represent the two vectors on a grid with a large scale (so that the angle between them is easy to measure). A scale of 1 unit : 2 cm will work well. The screens below show the *ClassWiz* operations, with lengths stored in x and y and the included angle stored in C, using ⇄x for convenience. Students should use a ruler and protractor to verify that these results seem correct.



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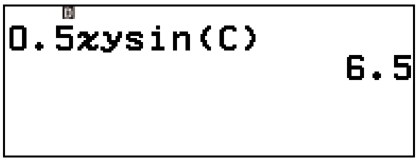
The following screen shows the area calculation, using the stored values:



These measurements are a little tedious, but provide students with a chance to see how *ClassWiz* makes various vector measurements and how these can be used. Make sure that students realise that storing the lengths and the angle as variables x, y and C respectively preserves their values to the precision available on *ClassWiz*, which would not be the case if they were simply recorded on paper and then used for a calculation.

Activity 4 explores further with two important vector operations. In this case, students should not be surprised to find that vectors **a** and **b** are not orthogonal (i.e. their dot product is non-zero), as the angle between them was measured in Activity 3. There are many possible vectors **c** orthogonal to **a**, but perhaps the easiest to find involve reversing the components and multiplying one of them by -1. Here are two possibilities:

$$\mathbf{c} = \begin{bmatrix} 1 \\ -3 \end{bmatrix} \text{ or } \mathbf{c} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

Scalar multiples of these will also be orthogonal to **a**. Students can check these results on *ClassWiz* to confirm that the dot product in these cases is zero and check on the graph that the vectors are perpendicular to each other.

Students may be surprised, however, to see that the third component of the cross product (i.e. 13) is the area of the parallelogram formed by the two vectors; Activity 3 determined that the area of the triangle – half of the parallelogram – was 6.5.

As the entire investigation has dealt only with one pair of vectors, students should repeat the activities using a different pair of vectors of their own choosing, as suggested by Activity 5.

27. Investigating equations

The suite of numerical equation solving tools in the Equation mode of *ClassWiz* includes linear equations, polynomial equations and a general purpose equation solver (which was used in *Investigation 21: Investigating population models*). In this investigation, only the first of these is used, to understand the solution of systems of simultaneous linear equations.

Activity 1 explores solutions to this system of two equations in two unknowns, x and y:

$$4x + 2y = 5$$
$$3x - 5y = 7$$

Make sure that students can see that the equations entered into *ClassWiz* are the same as these, even though the layout looks a little different. After the coefficients are entered, the solutions are readily obtained with EXE and EXE. Provide students with suitable graph paper so that they can plot the lines represented and see that the solution of the system matches the point of intersection of the lines. Encourage students to construct further systems and check their solutions (mentally, if possible), working with a partner to increase familiarity with the *ClassWiz* operations and confidence in their results.

Activity 2 highlights a problem (which might have arisen when students were examining their own systems of equations in Activity 1): sometimes there is no solution available. Ask students for likely reasons for this to be the case, and check with suitable graphs. Informally, expect that some students will notice that, while the first equation shows $4x + 7y$ to be equal to 11, the second equation suggests that $2(4x + 7y) = 8x + 14y = 13 \neq 22$, so that it is not possible to satisfy both equations at the same time. Equations like this are called *inconsistent*. Students who have learned about slopes of lines will see that the two equations represent lines with the same slope – that is, they are parallel – and so they will not have a point of intersection. A graph will illustrate the same problem too.

Changing the constant term in the second equation (to anything except 22) will result in inconsistent equations with no solution. Similarly, multiplying the x and y coefficients, but *not* the constant term by the same number (such as $12x + 21y = 30$) will lead to further examples of inconsistent equations.

Encourage students to invent their own systems of equations that have no solution and affirm their choices with *ClassWiz*.

Activity 3 also illustrates a situation in which a single point of intersection is not available. A careful check – or a graph – will make clear that the coefficients of one equation are three times those of the other equation, so that not only are the lines parallel, but they are identical. All the points on the first line are also on the second line, so that there are an infinite number of solutions. Any solution to the first equation will also be a solution to the second equation, so the system does not have a unique solution. These are known as *dependent* equations.

Encourage students to construct similar systems of equations with infinite solutions to understand how they come about and to recognise the nature of the problem.

More experienced students might be encouraged to explore analogous situations for systems of linear equations in three or four unknowns, which are also handled readily by *ClassWiz*.

28. Investigating Fibonacci spreadsheets

The *Spreadsheet* app on *ClassWiz* lends itself to investigating mathematical ideas, as spreadsheets are commonly used elsewhere to represent and then explore situations. In this investigation the spreadsheet is an ideal tool for investigating some of the celebrated mysteries of the celebrated Fibonacci Sequence.

Activity 1 is concerned with generating terms of the sequence which are defined recursively, with each term (after the first two) equal to the sum of the previous two terms. A spreadsheet is an excellent tool for constructing recursive relationships of that kind. The description of the activity contains most of the information needed to accomplish this task, and will be familiar to those accustomed to spreadsheets like Microsoft *Excel*. Students completely unfamiliar with spreadsheets might require a little more help from the teacher.

Encourage students to scroll their results, to check that they follow the requirements for obtaining successive terms of the sequence and to see how the formula A1 + A2 has been automatically adjusted for each row of the A column. (This is a consequence of *Fill Formula* being used, rather than *Fill Value*). Note that it is possible to scroll upwards from cell A1 as a faster way to get to cells at the bottom of the spreadsheet. As a check, students will find that the 30^{th} Fibonacci number is 832040.

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Although it is sometimes possible to have up to 45 rows (depending on other material in the spreadsheet), it is not necessary to do so for this investigation.

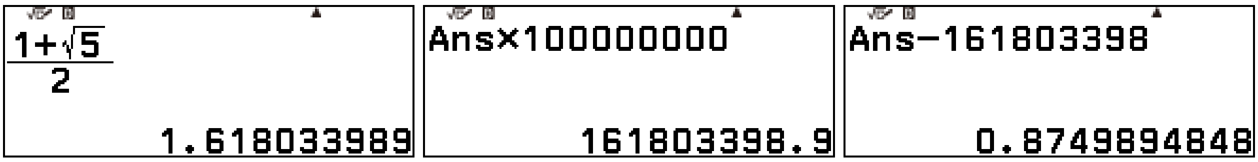
Activity 2 is concerned with the ratio of successive Fibonacci numbers, and also requires a formula, this time in the second (B) column. After entering and activating the formula, students will notice that all terms in column B from cell B13 downwards have the same contents: 1.618 . A discussion might reveal that the ratios are all the same, correct to three decimal places, but the cell is too small to show more decimal places. For most students, this convergent result is a surprise.

Activity 3 helps resolve the precision problem by showing the value of a cell rather than the formula that generated its contents, as explained and demonstrated. Once the values are shown, they are visible in the bottom of the screen (where the formula previously was). Many cells previously showing the same value now have different values, because of the extra precision. However, cell contents are again the same from the 25th row downwards: 1.618033989 (the limit of screen accuracy of *ClassWiz*) strongly suggesting that the ratio converges.

Some students might recognise this number as an approximation to the famous *Golden Ratio*:

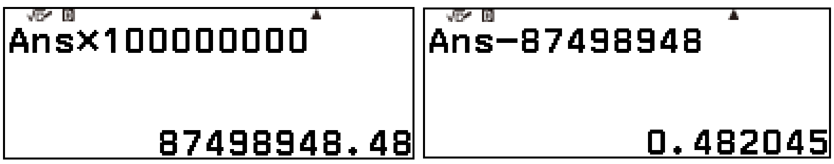
$$\varphi = \frac{1+\sqrt{5}}{2} \approx 1.618033989\ldots$$

To help them develop a sound understanding of the precision of devices like *ClassWiz*, students could investigate this remarkable number in Calculate mode (and will not lose their spreadsheet unless they turn the *ClassWiz* off). The number is firstly shown to nine places of decimals, but other decimal places stored are not immediately visible. A means of seeing further is shown below:



The screens above show how further stored decimal places can be revealed on *ClassWiz* by a process of multiplying a result by a large power of ten and then removing the integer part of the result to reveal the following decimals. So the approximation can be seen here to be 1.61803398874989484…

Continuing that process yields the number stored in *ClassWiz* to 22 places of decimals:



The Golden Ratio is an irrational number, with an infinite number of decimals. These processes show that *ClassWiz* has stored the result 1.6180339887498948482045 …to 22 places of decimals (the last place is uncertain, as it may have been rounded).

Interested students might like to check the spreadsheet result in this way to see that it is the Golden Ratio to only 9 places of decimals by the 25th term, although it has converged to the visible first nine places of decimals; further terms are needed to get more precision.

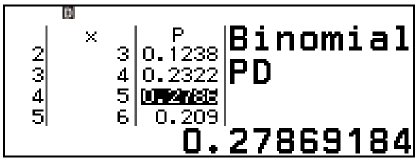
Activity 4 reveals another astonishing property of the Fibonacci sequence … if the first two terms are changed (and so all the following terms are also changed accordingly), the ratio of successive terms *still* approaches the Golden ratio! Encourage students to try some other changes to the first two terms … and the spreadsheet will then complete all the calculations needed to see what effect those changes have.

Students might be encouraged to look further online or elsewhere to find out about the Fibonacci Sequence and the Golden Ratio, often celebrated by mathematicians and others interested in aesthetics, and to use their *ClassWiz* to help with further explorations, for which a spreadsheet will be a helpful tool.

29. Investigating distributions

Probability distributions are important for dealing with many situations involving random processes. In this investigation, the Binomial and Poisson distributions will be explored.

Activity 1 provides both information about how to use the *Distribution* app as well as how binomial probabilities can be calculated from first principles. It is a useful exercise for students to complete such calculations, if only to appreciate the time saved by using *ClassWiz*. Both *List* and *Variable* are useful for different purposes, and students should become accustomed to each and to choosing the one that suits their needs. In this case, for the probability distribution (PD), the partial list of probabilities shown below reveals that the single most likely outcome is 5 successes, although 4 and 6 successes are almost as likely.



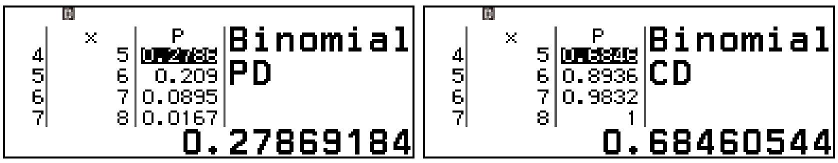
Activity 2 investigates the corresponding cumulative distribution (CD), which students should explore. Ask them to compare the PD and CD information: they should notice that

$$\text{Prob}\ (x \le 2) + \text{Prob}\ (x = 3) = \text{Prob}\ (x \le 3)$$

In this case, the data on the sheet gives

$$0.0498 + 0.1238 = 0.1736 \text{ (to 4 decimal places)}$$

Encourage students to look for various solutions, to understand the probabilities involved. For example, to see how likely Michael is to get 6 or more goals in 8 throws, there are two possibilities, shown below:



One solution is: $\text{Prob}\ (x \ge 6) = \text{Prob}\ (x = 6) + \text{Prob}\ (x = 7) + \text{Prob}\ (x = 8)$
$\approx 0.209 + 0.0895 + 0.0167$
≈ 0.315

An easier solution is: $\text{Prob}\ (x \ge 6) = 1 - \text{Prob}\ (x \le 5)$
$\approx 1 - 0.6846$
≈ 0.315

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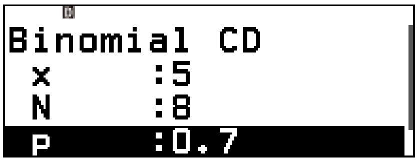
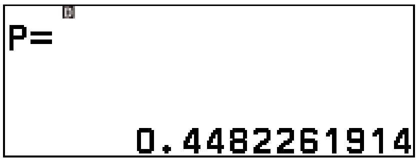
If Michael's success rate improves to 70% (i.e. $p = 0.7$), he is somewhat more likely to score 6 or more goals as the screens below suggest:

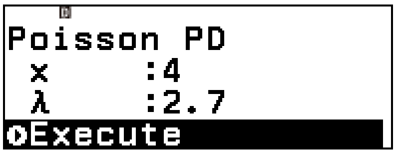


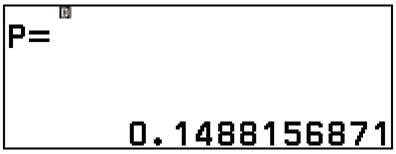


With the improved rate, Prob $(x \geq 6) = 1 - \text{Prob}\ (x \leq 5)$
$\approx 1 - 0.4482$
≈ 0.552

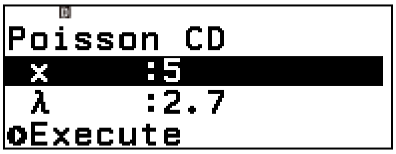
Encourage students to vary N and p and compare both CD and PD results to develop a good sense of how the binomial distribution works, as well as how it is supported by *ClassWiz*.

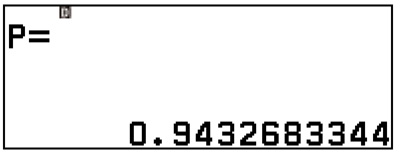
Activity 3 briefly examines a Poisson situation, for which there is only one parameter, λ, the average rate of occurrence of a random event. The app works similarly to that of the Binomial distribution. The first question asked in the investigation can be efficiently answered with the following screens (although others are also possible), showing that the probability of a cake having exactly four raisins is about 15%:





The second question can be most efficiently answered with these screens:





A cake having more than five raisins is the complement of having five or less raisins. So the probability is 1 – 0.943 or about 6%.

Again, encourage students to experiment and change the parameter λ to get a sense of the likelihood of different events.

30. Investigating curve fitting

The bivariate statistics capabilities of *ClassWiz* are useful for dealing with real-world data in which there are elements of random variability. For example, we seek a line of 'good fit' or even of 'best fit' to model data with a linear function. In this investigation, however, the aim is to fit a line or a curve *exactly* to data.

Of course, this is only possible with limited data: two points determine a line; three non-collinear points determine a parabola; two non-colinear points determine an exponential function, and so on.

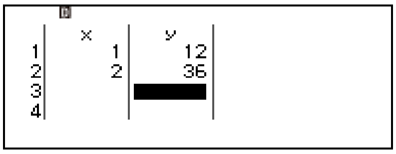
Activity 1 explores the case of a line determined by two points (not on a vertical line). Students are assumed to be familiar with Statistics mode. The regression results for a linear model indicate a correlation of $r = 1$, which of course never happens with real world data. *ClassWiz* can be used to determine other points on the line, using the predicted values (shown with ˆ). The example used in the investigation might just as easily be handled in other (analytic) ways, but this would not be the case if the points were awkward numbers, and not integers.

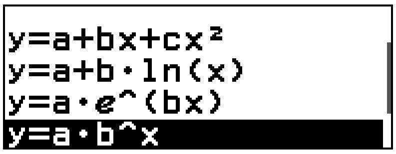
Activity 2 investigates a similar idea for three points which will determine a unique parabola (provided the points are not collinear). Again, points on the parabola can be predicted, as in the linear case, but if an x-value associated with a particular y-value is predicted, there are two solutions, due to the nature of the parabola.

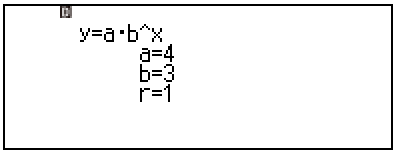
Make sure students are alert to the use of the coefficients in the parabola, given as $y = a + bx + cx^2$, as this is different from the general representation of $y = ax^2 + bx + c$ (which is used, for example, in Equation mode on *ClassWiz*).

The parabola through the three points (1,-3), (2,-3) and (3,-5) is given by a = -5, b = 3, c = -1 and is thus $y = -x^2 + 3x - 5$.

Activity 3 uses the same procedures with a different model, in this case the exponential model, requiring only two points to find the two parameters, *a* and *b*.







The screens show that the curve in this case is given by $a = 4$ and $b = 3$ and thus $y = 4 \times 3^x$. This is easily checked mentally, with $12 = 4 \times 3$ and $36 = 4 \times 3^2$.

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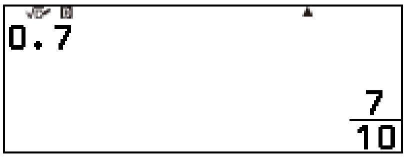
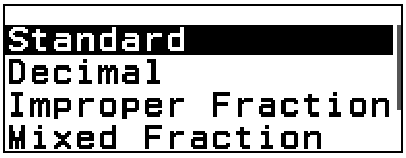
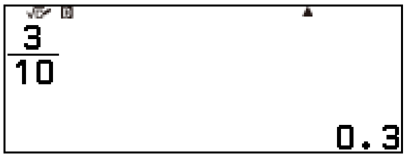
Investigating numbers 1

1. When a number is entered into your *ClassWiz*, it is usually displayed as a fraction. You can tap the FORMAT key to represent a number either as a decimal or a fraction in various ways; or tapping SHIFT before the EXE key will give a decimal result directly.







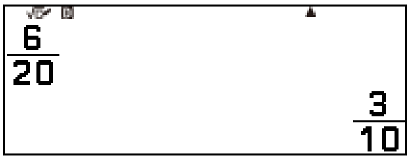
Enter some other small numbers like this, such as 0.9, 0.5 and $\frac{6}{10}$. Choose some others. Share your results with a partner. Can you explain what is happening? Write your conclusions here:

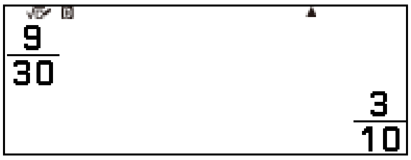
____

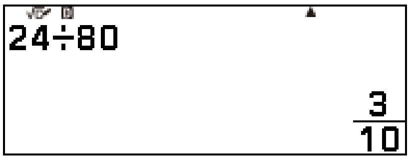
____

____

2. There are many other numbers with the same representation as $\frac{3}{10}$. Here are three of them:







Use your *ClassWiz* to find several others. How many can you find? Discuss this with your partner.

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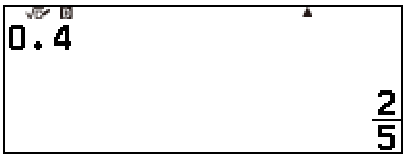
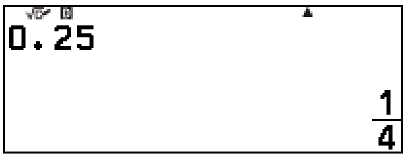
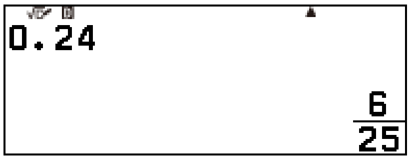
3. Kumi expected that 0.4 would be represented as four tenths, but *ClassWiz* represented it as two fifths. She found some other unexpected examples like this below.







Discuss these with your partner to explain what is happening. Make some more examples like these for yourself. Look for a pattern to predict the results before you tap the EXE key.

Share your results with a partner. Can you explain what is happening? Write your conclusions here:

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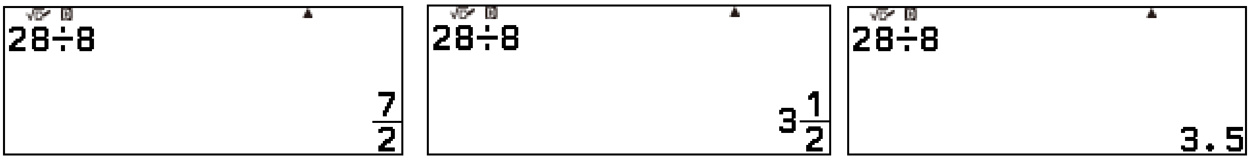
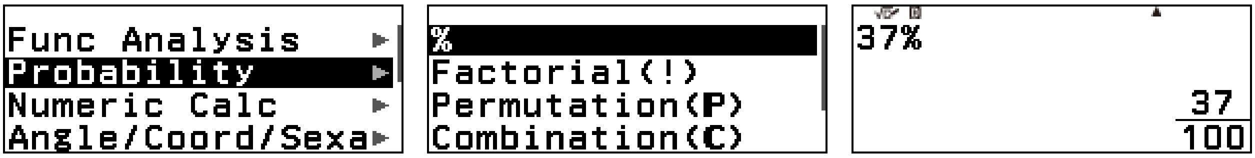
Investigating numbers 2

1. Numbers larger than one can be represented as fractions in two different ways, as a mixed fraction or an improper fraction. Tap (FORMAT) on your *ClassWiz* to switch between these two representations or to show them as decimals. Here is an example:



Discuss this example with your partner. Make several more examples of your own like these and predict the results *before using your calculator*. Test your predictions on your *ClassWiz*.

2. Sometimes numbers are represented as percentages, which are fractions with 100 in the denominator. To enter a percentage into *ClassWiz*, first enter the desired number, then select *Probability* from the catalog menu (CATALOG) and then select the % command, as shown below.



Enter some familiar percentages into your calculator to see how they are represented as fractions or as decimals. Compare your examples with a partner.

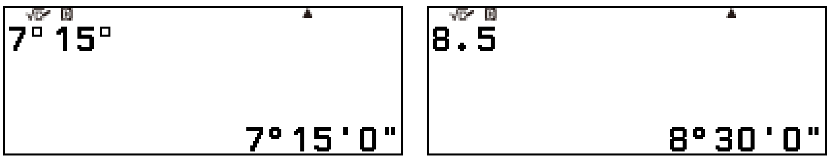
Arrange these numbers in order, from smallest to largest, using *ClassWiz* if necessary:

$$\frac{17}{25},\ 13 \div 20,\ 64\%,\ \frac{5}{8},\ 0.635,\ \frac{3}{5},\ 61\%,\ 33 \div 50$$

____

Check your order with a partner, and discuss how you reached your conclusions.

3. Clock times can be entered into *ClassWiz* using (SHIFT) (+) after each number. The time of 7:15 is shown below; notice that the result shows 0 seconds, even though seconds were not entered. A number can also be represented as a time using the *sexagesimal* measure command in (FORMAT), as shown in the second screen below.



You can use *ClassWiz* to calculate some times. [For example, if a movie starts at 3:35 pm and lasts for 1 hour 55 minutes, when will it finish?] Devise some problems like these for your *ClassWiz* and check your answers with your partner.

Use (FORMAT) to explore numbers represented as times to explore how *ClassWiz* uses fractions to deal with time questions. Write your conclusions here:

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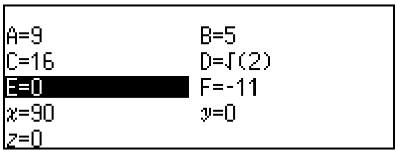
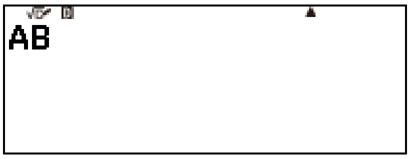
Investigating variables

1. *ClassWiz* includes some variables, and stores a single value for each. You can see the present values in your *ClassWiz* with the 🔲 key. You can change the value of a variable by highlighting it and then entering a new value followed by (EXE).

To use a stored variable in a calculation, start with the ⇧ key; for example ⇧ ④ recalls variable A and ⇧ ⑤ recalls variable B. Experiment with your *ClassWiz* to see how these operations work, as shown below.

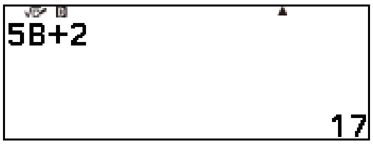


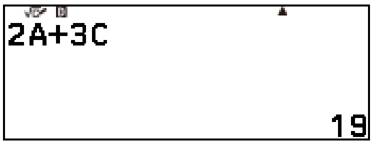
2. When $A = 7$ and $B = 6$, Wayan expected AB to be 76, while Nyoman expected AB to be 13 and Ketut expected AB to have the value of 42. Who is correct? Explain why the others are incorrect.

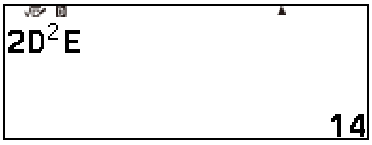


Discuss this with your partner. What would the three boys predict if the value of B was changed from 6 to 8? Use your *ClassWiz* to check.

3. More complicated expressions can be formed on *ClassWiz*. Here are some examples:



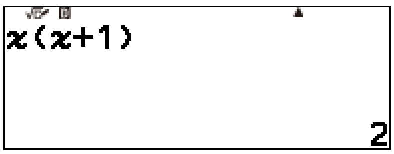


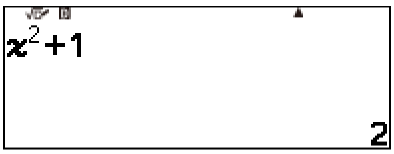


What values of the variables will produce these screens? (Is there only one answer?)

Check with your *ClassWiz* and explain to your partner what is happening.

4. The variable x is often used in mathematics. On *ClassWiz*, you can recall its value with ⇧ ⓪ or directly with the ⓧ key. When x has a stored value, expressions can be evaluated as shown below.





Discuss this example with your partner. For which values of x does $x(x + 1) = x^2 + 1$?

Make several more examples of your own like this one and predict the results *before using your ClassWiz*. Test your predictions on your *ClassWiz* and discuss them with your partner.

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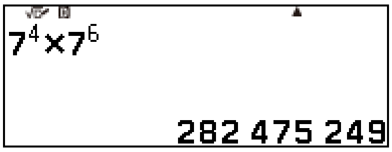
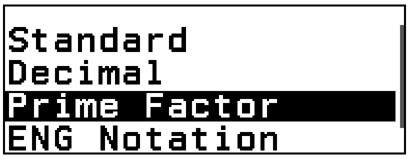
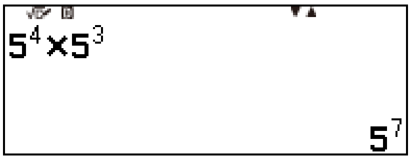
Investigating powers 1

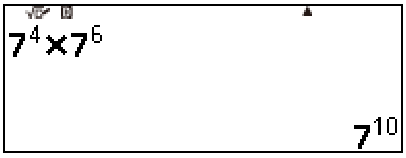
1. When powers of a whole number are multiplied together, the results are often large numbers:



When the *Prime Factor* command in the FORMAT menu is used *after the result is obtained*, prime factors of the result are displayed, as shown below. Try these two examples on your *ClassWiz*.







Then try several other examples like these for yourself. Write down the examples below and compare them with your partner:

Look for a pattern to predict the results before you find the factors.

Discuss with your partner what you notice. Summarise your observations here:

2. Write down what result you expect for 13^2 x 13^5, for 6^4 x 6^7 and for 17^3 x 17^6. Then check your predictions with *ClassWiz*:

Explain what is happening and *why* it is happening. Write your conclusions here:

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Investigating powers 2

1. Predict what will happen with these results as powers *before trying them on your ClassWiz*:

$3^4 \times 3^2 \times 3^5$ $\quad$ $15^3 \times 15^4$ $\quad$ $11^8 \div 11^3$

Explain any incorrect predictions:

Try some other examples of these three kinds until you can confidently predict the results.

2. Write down your predictions (as numbers raised to powers) for the result of the powers of powers shown below. Compare your predictions with your partner:

$(7^3)^2$ $\quad$ $(2^3)^7$ $\quad$ $(5^4)^3$

Then check your predictions on *ClassWiz* using (FORMAT) *Prime Factor*. Discuss with your partner what is happening, and explain the results to each other. Write down your explanations here:

Use your explanation to predict some fresh examples of your own like these. Test your predictions on your *ClassWiz*.

3. When you are confident that you understand how powers of powers are obtained, discuss you're your partner and write down what you expect to happen with these two examples, *before testing them on ClassWiz*.

$(6^3)^4$ $\quad$ $(4^3)^2$

Then check your predictions with *ClassWiz*. Discuss your reasons for your predictions with your partner, and explain the results to each other. Write down your explanations here:

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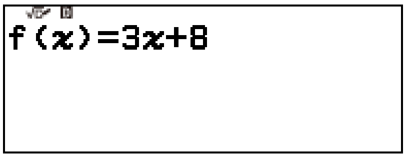
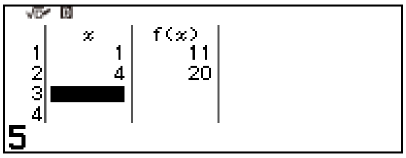
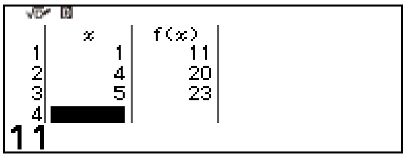
Investigating linear functions

1. *ClassWiz* can generate values for functions. The example below shows the linear function $f(x) = 3x + 8$ in Table mode. Use your *ClassWiz* to do this.





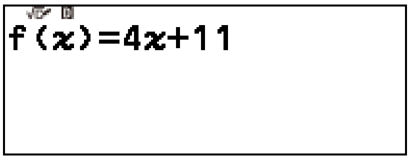


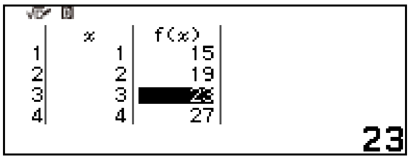
When a value for x is entered, followed by (EXE), the value of the function is shown by *ClassWiz*. Predict $f(11)$ before tapping (EXE). Try several more values for x, predicting the results first.

Explain how the values for $f(x)$ could also be predicted without using *ClassWiz*:

Now try a different linear function.

2. *ClassWiz* is very effective to generate tables of values for functions. The example below shows the linear function $f(x) = 4x + 11$ in Table mode.

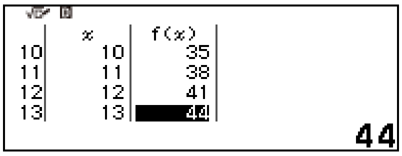


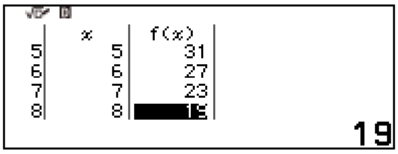


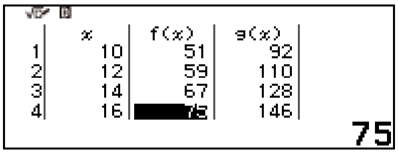
Look carefully at the pattern of values in the table. What will be the next numbers? Explain how you know:

Scroll down with (⌄) and (⏬) to check your predictions.

3. Once you understand the pattern of values of a function, it is possible to identify the function by analysing the values.







Study carefully the three screens above. Which four linear functions are involved?

Use your *ClassWiz* to make these three screens to check your answers. Then make up some more examples of this kind to share with a partner – see if they can find your functions.

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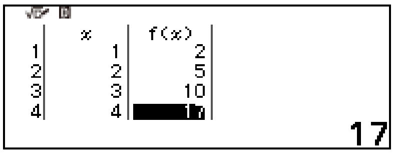
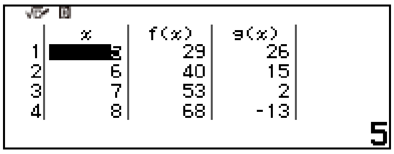
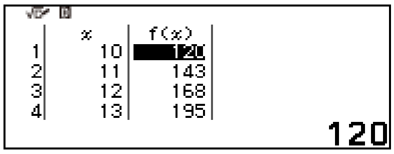
Investigating quadratic functions 1

1. Quadratic functions result in different kinds of patterns than linear functions; the difference between successive values is not always the same. The first screen below shows some values for the quadratic function, $f(x) = x^2 + 1$.







Look carefully at the patterns of values in the tables. What will be the next numbers in each case?

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Use the *ClassWiz* to test your answers, with ⌄ and ⩔.
What are the other three functions tabulated?

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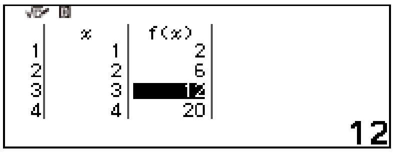
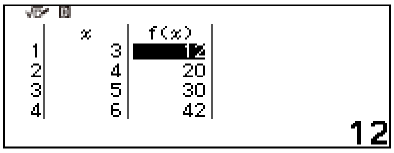
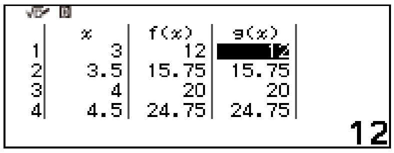
2. James was investigating a table for $f(x) = x^2 + x$ in the first screen below, while Elly was investigating $f(x) = x(x + 1)$ in the second screen. The results seemed similar, so they chose to investigate them together, with $f(x) = x^2 + x$ and $g(x) = x(x + 1)$ in the third screen.



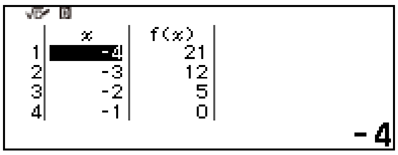


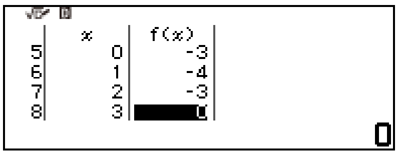


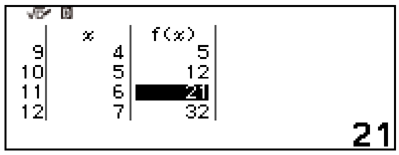
Use *ClassWiz* to change the table settings to investigate with a partner. Explain what you notice:

____

3. Takashi was using his *ClassWiz* to help sketch a graph of $f(x) = x^2 - 2x - 3$ on graph paper. Here are some of the tables he made:







Study these tables to imagine the graph: What shape should it have? Where does it cross the x-axis? Does it have a maximum point? Does it have a minimum point? Is it symmetrical?

____

____

Draw and label some axes on a fresh piece of paper and sketch the graph with a pencil, using the table values to help. Check your sketch with your partner.

Then use your *ClassWiz* to help you to graph a different quadratic function.

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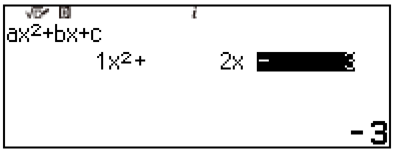
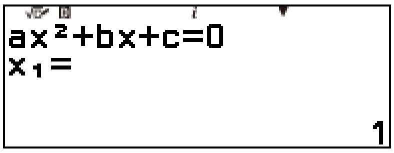
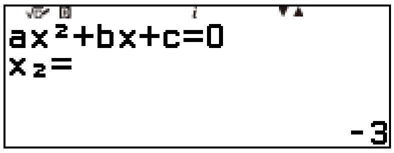
Investigating quadratic functions 2

1. It is often helpful to graph quadratic functions, or to visualise their parabolic graphs, to understand them well. Consider the example of $f(x) = x^2 + 2x - 3$. *ClassWiz* can provide useful information for graphing, by considering the associated quadratic equation of $x^2 + 2x - 3 = 0$. Solutions to this equation are roots of the function. Firstly, solve the equation in *Equation* mode:



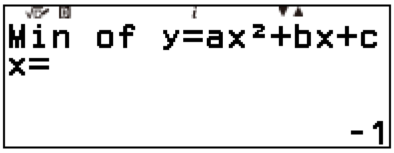


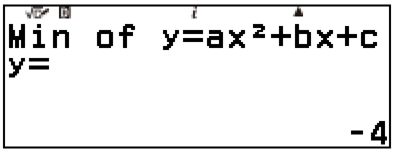


After selecting the (*Polynomial*) equation, tap (EXE) twice to get the solutions. Check (mentally) that these two solutions satisfy $f(x) = 0$. Use some grid paper to start sketching the function.

Where does the graph cross the x-axis? ____

Tap (EXE) twice more to see the turning point of the parabola:

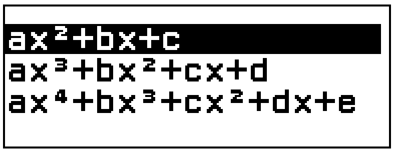


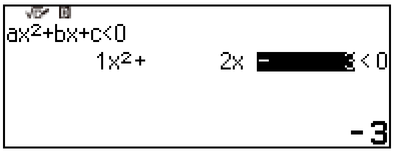


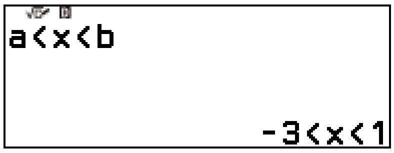
What are the coordinates of the turning point? Describe the shape of the parabola:

____

2. Further, related, information about a quadratic function can be obtained from *Inequality* mode. Choose the quadratic function $ax^2 + bx + c$, select one of the four possible inequalities and enter the coefficients. Tap (EXE) to solve the inequality. For $f(x) = x^2 + 2x - 3$, here is one possibility:

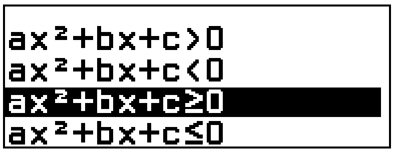


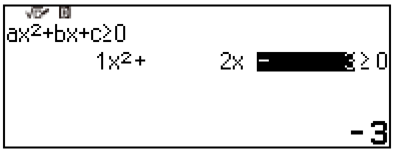


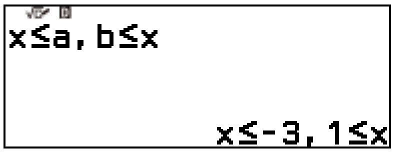


For which values of x is the function negative? (i.e. $f(x) < 0$)? ______________________

Here is another possibility, this time showing values of x for which the function is non-negative:







For which values of x is the function not negative? (i.e. $f(x) \geq 0$)? ___________________

Explain why it is not necessary to use *both* these inequalities:

____

You should by now have enough information to make a good sketch of the function. Complete your sketch and check it with your partner.

3. Use your *ClassWiz* to help graph some other quadratic functions in this way.

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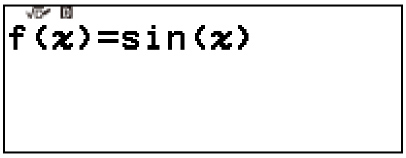
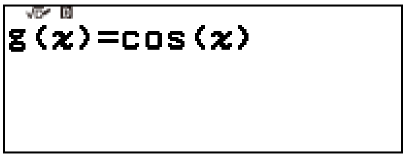
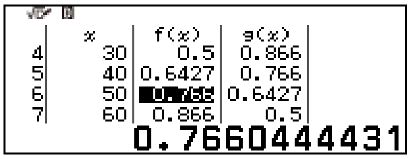
Investigating trigonometry

1. There are some interesting relationships between sines and cosines of angles that can be seen by studying their values in tables. In the example below, *ClassWiz* has been set to degrees, and the tables show angles in the first quadrant ($0^o \le x \le 90^o$). Use your *ClassWiz* to scroll and study carefully these tables.







Notice in the third screen that sin 50º = cos 40º and that sin 30º = cos 60º. Scroll the table on your *ClassWiz* to find some more examples of pairs of values that are the same.

Record your results on paper and discuss them with your partner.

____

Check if the same relationship holds for other values by changing the *Step* in the *Table Range* to another value, such as 5º, so that there are more values displayed. Describe the pattern you see:

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Explain *why* the sine and cosine values are related in this way. A right triangle or a unit circle may be helpful for this.

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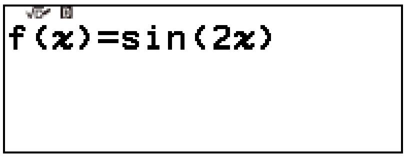
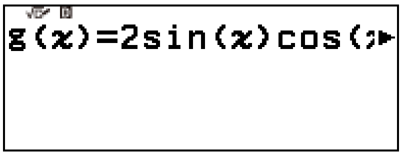
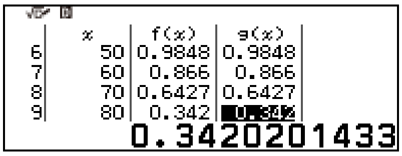
2. Trigonometric *identities* are important, as they show general relationships. A good example is $\sin 2x = 2\sin x \cos x$. To appreciate what this means, enter each side of the identity into a table, as shown in the screens below.







Check for yourself that this relationship is *always* true by changing the table values: use different limits, use a different step, or change some x-values in the table directly. Make sure that you check positive and negative values, large and small.

Check some other possible identities in this way, like those below.

$$\tan x = \frac{\sin x}{\cos x} \qquad \tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$\cos 2x = \cos^2 x - \sin^2 x \qquad \cos(90^\circ + x) = \sin x$$

Use your *ClassWiz* to check which of these – if any – are *not* identities.

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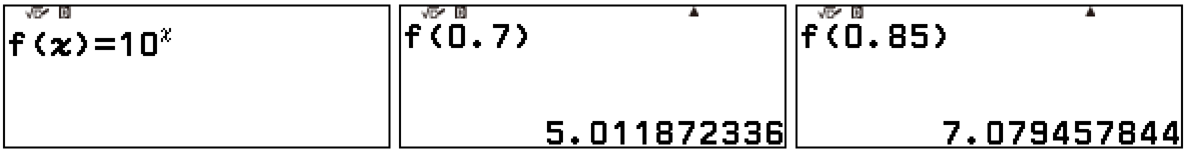
Investigating powers 3

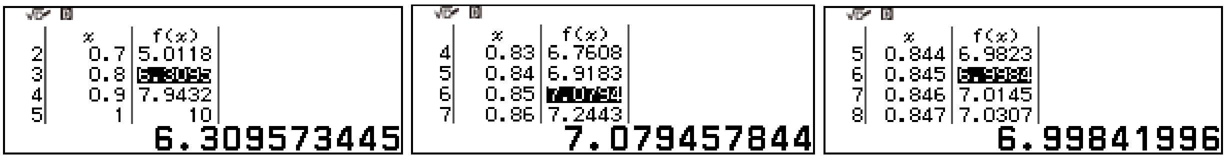
1. Integral powers of 10 such as 1, 2 and 4 lead to whole numbers such as 10, 100 and 10 000. But this is not the case for most other powers of 10. For example, what power of 10 is equal to 7? To investigate this question, Manuel defined a power function using ⓕ(x) as below. He realised that the power must be less than 1, as $10^1 = 10$, so he first guessed $x = 0.7$.



To refine his guess efficiently, he tapped ⓛ ⓛ and edited the x-value. His next guess of $x = 0.85$ was a bit too large, as the screen shows. Repeat this process to get as close as you can to the x-value that gives $10^x = 7$. Check your result with others:

---

2. Spiro decided instead to make a table to explore $f(x) = 10^x$ to find the power giving $f(x) = 7$.



Notice carefully that each table uses a smaller *Step*. Continue Spiro's approach to get as close as you can to finding the desired result. Compare your work with your partner.

Compare your result with Activity 1. Which method do you prefer? Why?

---

3. Choose your own procedure (that suggested by Activity 1 or by Activity 2) to find as closely as you can the power of 10 that will result in 123 and the power of 3 that results in 7. Check your answers with your partner:

---

4. This idea is not restricted to whole numbers. In fact, you can use similar procedures to find the power of ***e*** that results in 7. (The number ***e*** is very important in mathematics and so is on your *ClassWiz* keyboard with ⓢ ⑧, as shown below. You will learn more about it later in your studies).

It seems that the power needed is less than 2. Experiment to find x so that $e^x = 7$. Check with others.

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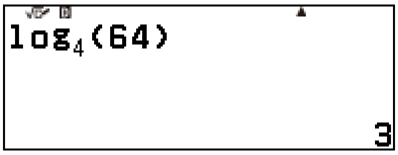
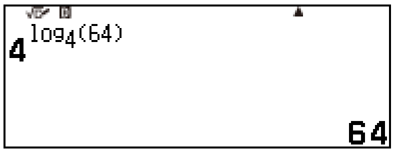
Investigating logarithms

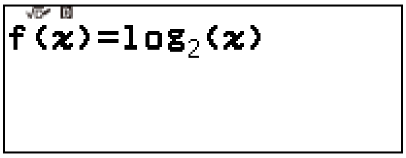
1. The $\log_{\square}\square$ command finds the logarithm of a number to a particular base. The logarithm is the power of the base needed to produce the number. Study carefully these three screens showing that the logarithm to base 4 of 64 is 3.

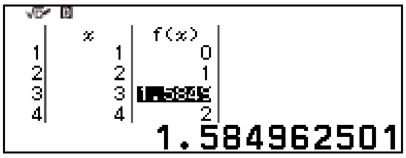


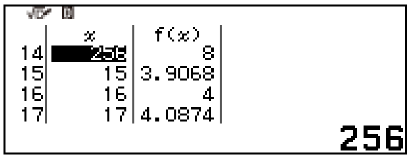


Use your *ClassWiz* to make several other sets of three screens like these. Use a variety of bases. Use positive and negative powers. Record your results on paper and discuss them with your partner.

2. Most logarithms are not whole numbers. A table of logarithms can show many examples, as shown below. (Notice in the third screen that you can replace any number in the x column with a number of your own choice. In this case, 14 was replaced by 256.)







Explore this table by scrolling, changing the x-numbers or changing the limits of the table.

Larger numbers have larger logarithms. Explain why this is the case.
Which numbers have logarithms that are whole numbers? Explain why.

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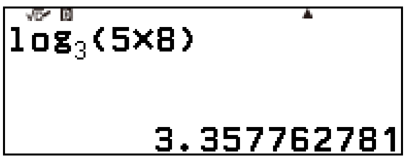
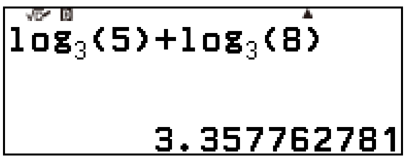
Make some tables with logarithms to a different base, to check that your explanations still apply; check your observations with your partner. Discuss why, *for all bases*, the logarithm of the base itself equals 1 and the logarithm of 1 equals zero. Write your explanation here:

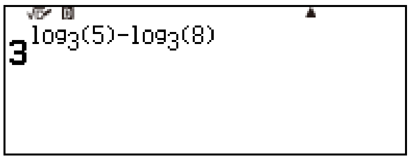
____

3. Study the first two screens below to see that the logarithm of a product of two numbers is the sum of their logarithms, thus connecting multiplication and addition.

Before using your ClassWiz to check, what result do you expect for the third screen?____________







Make several more examples of your own like these, using different numbers and different bases.

4. Discuss your Activity 3 examples with your partner. Explain how to use this property of logarithms to find the logarithm of a *power*. For example, how does the logarithm of 35^7 compare with the logarithm of 35?

____

Check your answer with *ClassWiz*, using several different bases.

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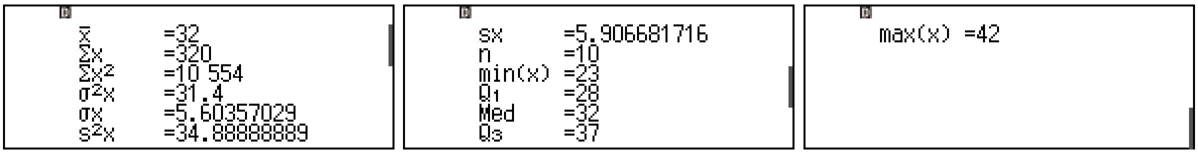
Investigating univariate data

1. To report data to the government, a Principal counted the number of students in each of the ten classes of her school. Here are the results:

25, 28, 32, 37, 35, 29, 23, 37, 32, 42

She used a *ClassWiz* to analyse these data. Check her results with your *ClassWiz*.



The Principal then realised that she needed to include all of the class occupants, and so needed to add one person (the teacher) to each class size. What effect will this change have on these statistics?

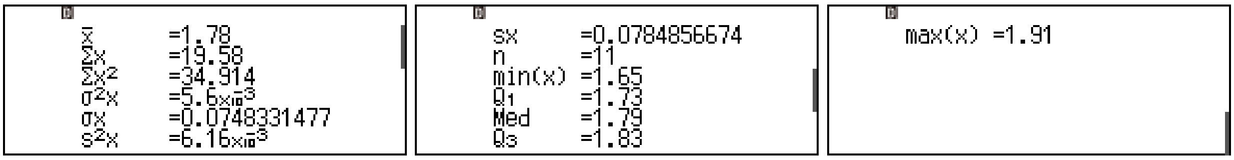
Edit the data in your *ClassWiz* and analyse the corrected data to check your prediction.

Discuss with your partner the effect of adding 2 (instead of 1) to each number, to also include a teacher assistant in each classroom. Check your prediction with the *ClassWiz*.

2. A manager measured the heights (metres) of players in a football team with the following results:

1.65, 1.78, 1.82, 1.66, 1.85, 1.81, 1.79, 1.75, 1.91, 1.83, 1.73

These data were to be used to include in the team's annual magazine, and a *ClassWiz* was used to summarize them. Check the manager's results with your *ClassWiz*.



Heights of players in other teams were measured in centimetres, however, not metres. So each of the heights above needed to be multiplied by 100. What effect will this change have on the statistics shown here?

Enter the corrected data into your *ClassWiz* and analyse them to check your prediction.

3. What is the effect of multiplying each number by 10 (instead of by 100)? Check your prediction with *ClassWiz*. Discuss with your partner.

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Investigating arithmetic sequences

1. A sequence is a set of numbers in order. You can start a sequence on *ClassWiz* with any number, followed by ⓔⓧⓔ. Then use *ClassWiz* to generate the next term. In the example below, we started with 9 and then used ⊕ ④ ⓔⓧⓔ to make the second term four more than the first. (Notice that *ClassWiz* interprets this as adding four to the previous answer, called *Ans*.) Sequences of this kind are called *arithmetic* sequences. In this case, the *common difference* is 4.

Now tap ⓔⓧⓔ several times (to repeat the same operation) to get further terms: 9, 13, 17, …

Use *ClassWiz* to generate the arithmetic sequence 5, 14, 23, … What is the tenth term?

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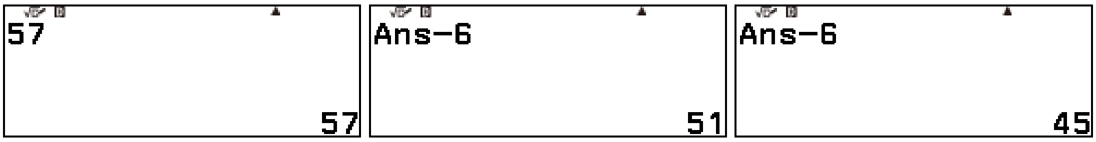
2. Arithmetic sequences can also get smaller with each successive term. Here is an example, starting with 57:



Investigate the arithmetic sequence 57, 51, 45, … on your *ClassWiz*. What is the common difference? What is the fifth term? How many terms are positive?

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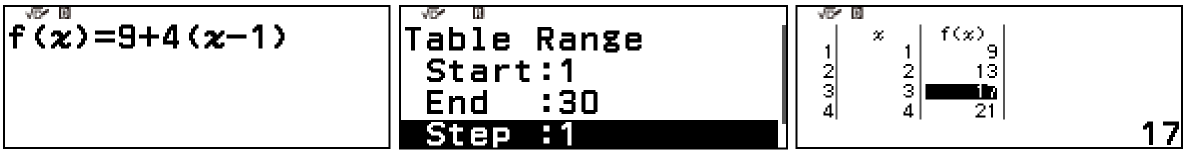
3. You can also make an arithmetic sequence using a table. The x value refers to the term number and the function value gives the terms of the sequence. The third term of the sequence below is 17.



Note that this is the same sequence as that described in Activity 1. Why is the table a better way of showing the sequence? Explain the meaning of 9 and 4 and $x - 1$ in the function definition.

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Use the function directly to find the 100^{th} term of this sequence. Check with your partner.

____

4. Which function gives the arithmetic sequence 57, 51, 46, …? Check with a *ClassWiz* table.

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Investigating geometric sequences

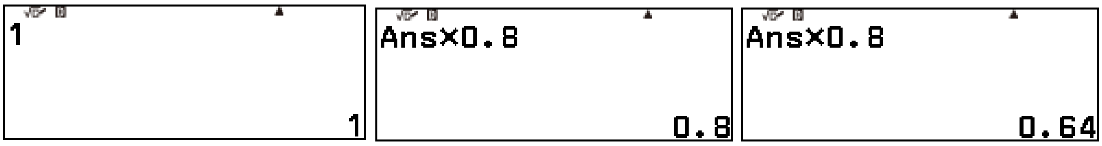
1. In a geometric sequence, each term is a multiple (called the *common ratio*) of the previous term. You can generate a geometric sequence on *ClassWiz* in a similar way to generating an arithmetic sequence. In the example below, the first term is 4 and the common ratio is 3. Starting with 4, the second term is found with [×] [3] [EXE] while later terms are produced by tapping [EXE] in succession:

Geometric sequences like 4, 12, 36, … have large terms quickly, when the common ratio is large.

Tap [EXE] in succession for this sequence, counting carefully the number of terms. What is the seventh term? Which is the first term larger than 50 million? Check with your partner.

---

2. When the common ratio is less than 1, geometric sequences get smaller with each successive term. For example, if a rubber ball bounces to 80% of its previous height each time, a geometric sequence can be used to describe its height after every bounce, being dropped at first from one metre high (using [SHIFT] [EXE] instead of [EXE] to give decimal values for each term):



Investigate this geometric sequence on your *ClassWiz*. How high will the fifth bounce be? After how many bounces will the ball bounce less than 10 cm? What happens eventually?

---

3. Geometric sequences can be generated with a table. Study carefully the example below, showing a city population (in millions) that is growing by 2% every year, starting at 8 million people.

Explain the meaning of each of the elements 8, 1.02 and $x-1$ in the function definition.

The table suggests that the city population in the fourth year will be 8 489 664. Discuss with your partner how likely this is to be accurate. About when might the city population reach 10 million?

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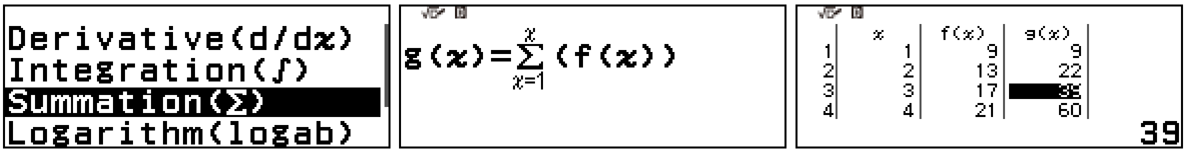
Investigating series

1. A *series* is a special kind of sequence, obtained by adding successive terms of a sequence. For example, the arithmetic sequence 9, 13, 17, … has an associated arithmetic series 9, 22, 39, …

Define the sequence itself on *ClassWiz* with $f(x) = 9 + 4(x - 1)$, for $x = 1, 2, 3, \ldots$

ClassWiz can then be used to find terms of a series, using the *Summation* (Σ) command in the *Functional Analysis* menu shown in ⊜. The screens below show how to do this with a second function *g*. Check that the terms shown are correct for both sequence and series.



Use the table to find the sixth term of the series. Explain how you know the result is correct:

Now change the definition of $f(x)$ on *ClassWiz* to evaluate the series for the sequence 7, 12, 17, … Check that the fifth term of your series is 85.

2. A frog in the centre of a circle of radius 2 m jumps 1 m towards the edge. On the next jump, it jumps half as far as the previous jump. If it continues like this, what will happen?

The distance jumped each time (in metres) is a sequence: $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

Use your *ClassWiz* to define a suitable sequence and then its associated series.

How far will the frog have jumped after twenty jumps? … after fifty jumps?

When will the frog escape the circle? Discuss this with a partner.

3. Cans of soup are stacked high in a supermarket display like this:
(Only the top three rows of the stack are showing.)

If there are thirteen rows of cans in the stack, how many cans are there altogether?

If another stack like this has 210 cans, how many rows high is the stack?

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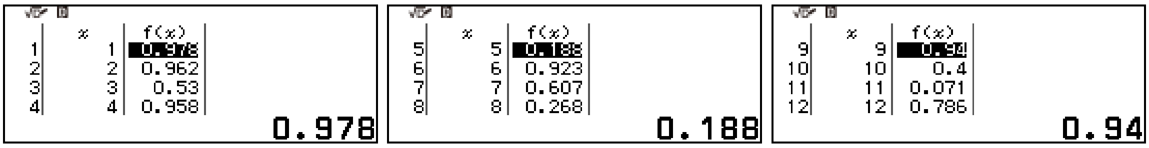
Investigating randomness

1. When *ClassWiz* is used to generate 3-digit random numbers using the *Random Number* command in the *Probability* catalog , the numbers will be uniformly spread between 0 and 1. So, about half the numbers should be less than 0.5. The tables below were generated using $f(x)$ = Ran#. In this case, four of the twelve numbers were less than 0.5.



Use your *ClassWiz* to generate 30 random numbers in this way and check to see how many are less than 0.5. Compare your results with others.

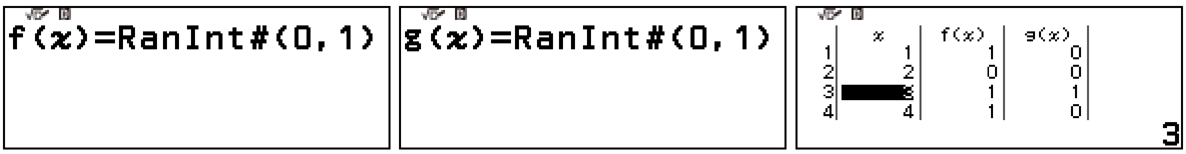
How many of 30 random numbers would you expect to be less than 0.3?

Generate a fresh set of 30 random numbers and check your prediction. Check with others too.

2. You may have noticed that, when a 3-digit random number ends in 0, it is shown as a 2-digit number. (The third random number above is an example, showing 0.53 instead of 0.530 .) How many 2-digit numbers would you expect in 20 random numbers? Explain why:

Use *ClassWiz* to test your predictions, by generating and testing several sets of 20 random numbers.

3. *ClassWiz* can simulate tossing a fair coin with the function *RanInt*(0,1). If the result is 0, we will regard it as a tail, with 1 representing a head. To simulate tossing a pair of coins, use both functions in the table, as shown below.



In this case, only one of the four tosses (the third one) shows a pair of heads. How often would you expect to get a pair of heads? Explain why:

Test your prediction by simulating pairs of coin tosses like this. Compare your results with others.

How likely are the other results? (a pair of tails or a head and a tail)?

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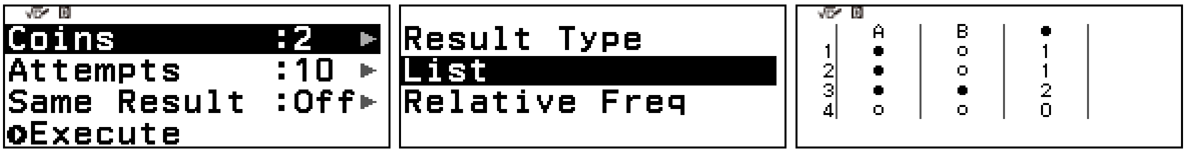
Investigating coin tosses

1. *Math Box* lets you simulate and study coin tossing efficiently with *Coin Toss*. Here is an illustration, showing two fair coins tossed ten times, and examining the results in a List:



The first screen of tosses shows the result for the two coins A and B and the total number of heads; heads are shown with a dark circle, tails with a light circle. Only the third toss shows two heads. The next set of tosses can be shown by tapping ⊻. (Your tosses will differ from these.)

Compare your results with others. How frequent are the possible results? (0, 1 or 2 heads)?

Tap ↺ ↺ to generate more tosses or to change the number of attempts.
How consistent are the results obtained?

2. To see what happens in the long run, much more data are needed. *Coin Toss* allows for up to 250 tosses of the two fair coins to be simulated, as shown below.

Choosing *Relative Frequency* instead of *List* instructs *ClassWiz* to summarise the results efficiently. In the illustration above, 61 of the 250 tosses resulted in two heads, a relative frequency of 61/250 (or 0.244). In this case, 50.8% of tosses resulted in a head and a tail.

Try this for yourself a few times. Compare your results with others. What conclusions can you draw from your results?

Explain why some results are more likely than others.

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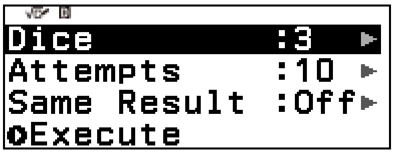
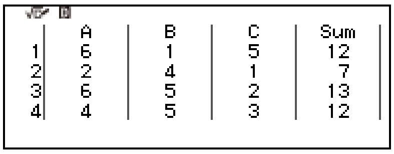
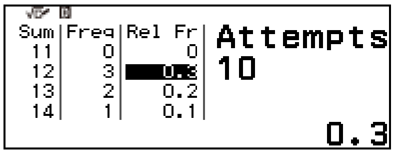
Investigating dice rolls

1. Rolling fair dice can be simulated with *Dice Roll* in *Math Box*. You can roll up to three dice and look at a *List* of the results or a summary in the form of a *Relative Frequency* distribution. The screens below show settings for ten rolls of three dice, with rolls represented by A, B and C.







The list shown in the second screen and the relative frequency distribution shown in the third screen represent the same simulated data in two different ways. (Use ⮌ to switch between the two representations). What are the smallest and largest possible sums of the three dice?

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Repeat this experiment a few times with a larger number of attempts, such as a hundred, instead of ten. Compare your answers with others, describing the distribution of sums of the three dice.

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Check your impressions with larger numbers of attempts (maximum 250). Which sums consistently seem most likely to appear? Check your results with others.

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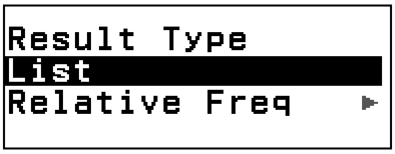
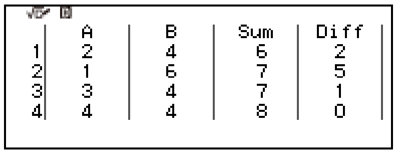
2. A game involves rolling a pair of fair six-sided dice and finding the difference between the scores on the two dice. What score differences are likely to happen in this game? The dice rolls, labelled A and B by *ClassWiz*, are shown below.





In this case only ten rolls were simulated and a *List* of results shown; both sums and differences of the A and B dice are shown. Which difference would you think is *least* likely to occur? Why?

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Compare your answers with others. Complete a number of simulations to check your thinking. After a few simulations, generate a larger data set, and use *Relative Frequency* to summarise your difference results. Do this several times and discuss what you notice with others.

What happens in the long run? E.g., which differences are most or least likely? Explain why these results might be expected:

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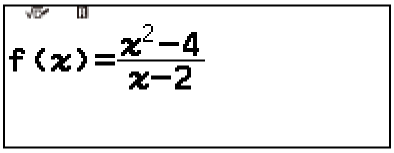
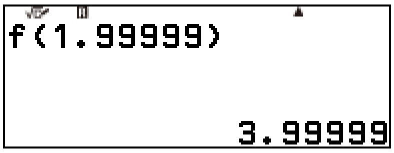
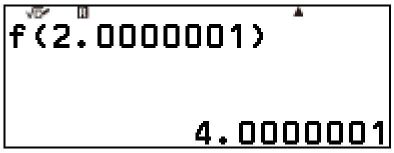
Investigating limits 1

1. To examine a limit numerically, evaluate the expression on *ClassWiz* for values that are closer and closer to the limiting value. For example, consider the $\lim_{x \to 2} \frac{x^2-4}{x-2}$; define a function using ⓕ⒳ and then check values using ⓕ⒳ again, increasingly close to $x = 2$.







Notice that this function is indeterminate at $x = 2$; i.e. $f(2)$ is undefined. Use a succession of values, getting closer and closer to the limiting value at $x = 2$. How close can you get? Discuss this with a partner: what is the limiting value?

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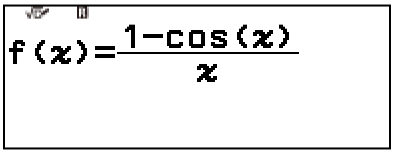
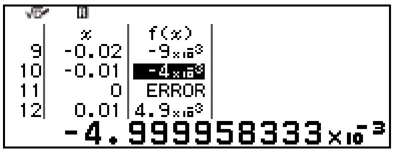
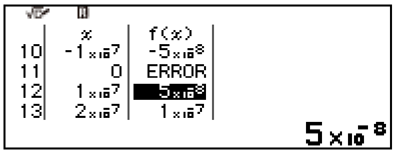
2. Rather than studying several values in succession, another approach is to use a table. For example, to investigate $\lim_{\theta \to 0} \frac{1-\cos\theta}{\theta}$, start by defining a suitable function and then make tables with the value of the variable increasingly close to the value sought. (Set *ClassWiz* angle unit to radians.)







The function is undefined at $\theta = 0$, but how close can you get? What seems to be the limiting value? Check your conclusions with a partner.

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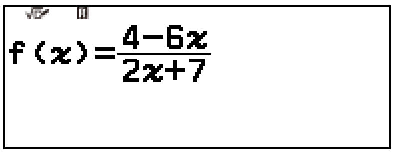
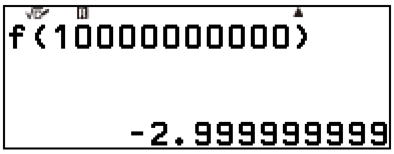
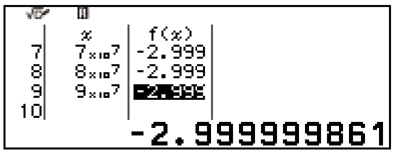
3. Limits to infinity can be explored in similar ways, checking values of an expression as the variable has larger and larger values. Consider $\lim_{x \to \infty} \frac{4-6x}{2x+7}$. With a partner, try each of the two approaches shown on this sheet to evaluate the limit. (Which approach do you prefer? Why?)







Try increasingly large values for x. What seems to be the limiting value in this case? Explain why.

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Invent several more examples of your own like this and predict the results *before using your calculator*. Test your predictions on your *ClassWiz*.

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Investigating limits 2

1. Limits of sequences and series are often very interesting. A famous series adds reciprocals of successive factorials of integers (starting with 0):

$$\frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\cdots$$

You can investigate this series on *ClassWiz*, by defining the sequence as a function $f(x)$ and the series as a related function $g(x)$, as shown below. [Note that the factorial command (!) and the summation command (Σ) are both available via ⊜ in the *Probability* and *Func Analysis* menus.]

$$f(x)=\frac{1}{x!} \qquad g(x)=\sum_{x=0}^{x}(f(x))$$

Examine this sequence and its associated series, either by finding some values, or by using tables, or both of these. What is the limit of the sequence? Of the series? Discuss these questions with a partner. Write your conclusions here:

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2. Compare the apparent limit of this sequence with that of the series in Activity 1. How are they similar? How are they different?

$$f(x)=\left(1+\frac{1}{x}\right)^{x}$$

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3. Define a sequence of reciprocals of positive integers, and its associated series (called the *Harmonic Series*) as shown below.

$$f(x)=\frac{1}{x} \qquad g(x)=\sum_{x=1}^{x}(f(x))$$

Examine this sequence and series using *ClassWiz*. Do either of them have limits? Write your conclusions here, after discussing these questions with your partner:

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Investigating population models

1. Mathematics can help with modelling, to understand the world population better and to make predictions. For example, according to https://www.worldometers.info , the world's population in 1965 was 3 334 533 703 and the annual world population growth rate was 2.03%.

Use an exponential model on your *ClassWiz* to predict the world population at some later times, such as in 1985, 2000, this year or near the middle of the 21st century, making reasonable assumptions about the annual growth rate. How well would your model have predicted the world population in 1985, which was 4 868 943 465?

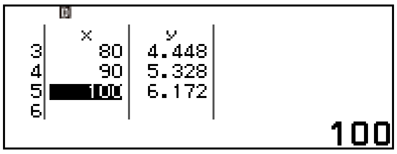
When would your model expect the world population to reach 10 billion people? Compare your methods with others and your results with recent world population data.

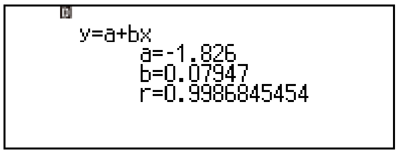
---

2. Another approach to modelling involves predicting from known historical data (assuming they are a reliable guide to the future). Here are some recent world population data up to year 2000 from *Worldometers*, shown in billions of people:

Year	1960	1970	1980	1990	2000
Population (Bn)	3.015	3.695	4.448	5.328	6.172

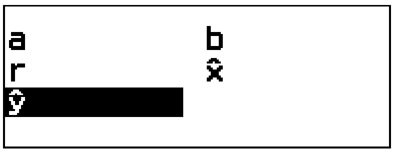
Enter these data into *ClassWiz*. Enter Statistics mode and entering data list-wise, followed each time by EXE. (Notice that years are given as 2-digit numbers, so 2000 is shown as 100.)

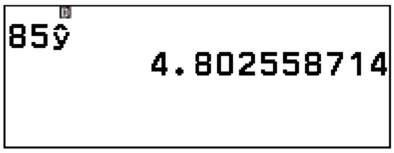




After entering data, tap OK and then choose *Reg Results* to see how well various models fit the data. The screen above shows a choice of a linear model $y = a + bx$. Discuss this result with a partner:

To make a prediction for x or for y, choose *Statistics Calc* and select a model. Use the ⌂ to select *Statistics* and then *Regression*. Enter the value first and then enter the relevant estimator, as shown





Different models use different assumptions to make different predictions. The above prediction for 1985 came from an exponential model, $y = ab^x$. Explore how well various models predict today's population. Compare your results with Activity 1. Discuss with your partner and with others.

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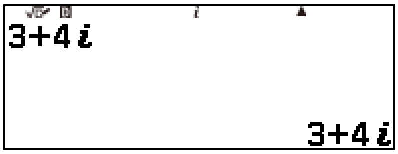
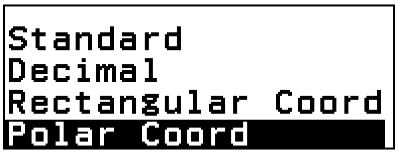
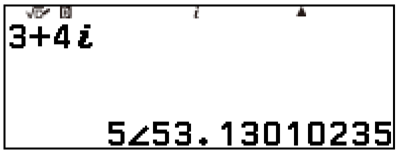
Investigating complex numbers

1. While real numbers can be graphed on a number line, complex numbers are often represented geometrically as points on a complex plane, with a real axis (x) and an imaginary axis (y). In complex mode (shown by the small i on the top of the screen), the symbol for i is obtained on *ClassWiz* with ⇧ ⑨. Check to see how the FORMAT menu allows both representations to be seen.

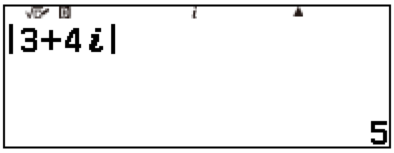


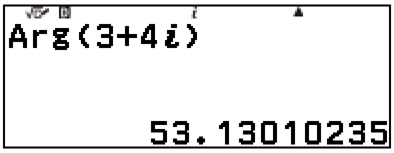




On an *Argand diagram*, plot some complex numbers and use a ruler and protractor to verify that the polar coordinates seem correct. How are conjugate numbers related when graphed? Explore what happens to its graphed point if a complex number is doubled. Which complex numbers will be graphed in the third quadrant? Compare your answers with your partner.

The modulus, or length, of a complex number can be found directly using the absolute value function in CATALOG in *Numeric Calc*, while the argument can be found in CATALOG in *Complex*, as below:





Use your graph to explain how *ClassWiz* determines the modulus and absolute value.

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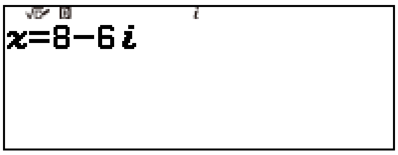
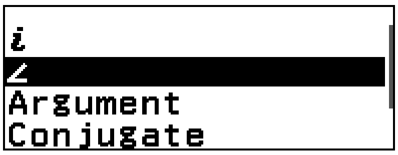
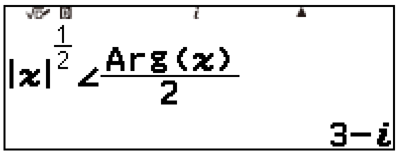
2. Although *ClassWiz* can find integral powers of complex numbers, it cannot determine fractional powers or roots directly. So it is necessary to use the wonderful *de Moivre Theorem* for these, using polar form for the numbers; in this case, r is the length and θ the argument of the number:

$$[r,\theta]^n = [r^n, n\theta]$$

For example, to find $\sqrt{8-6i}$, use $n = \frac{1}{2}$. Study the *ClassWiz* screens below. For efficiency, the complex number has been stored as x in the variable menu with VARIABLE. The angle symbol and command for Argument can be found in CATALOG in *Complex*.







Check by squaring that $\sqrt{8-6i} = 3 - i$. What is the other square root?

Use de Moivre's Theorem to find some other square roots, such as $\sqrt{32+24i}$ and $\sqrt{3+4i}$. Plot them on an Argand Diagram. What do you notice? Compare your observations with your partner.

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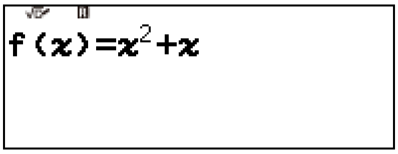
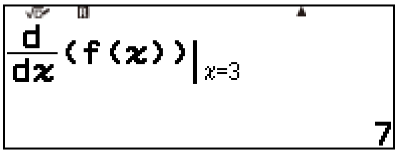
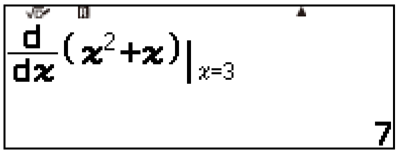
Investigating derivatives

1. The derivative of a function at a point describes the rate of change of the function at that point. If the function is graphed, the derivative will be the slope of the graph at that point. *ClassWiz* allows you to find the derivative of any function (of x) at a point, using the *Derivative (d/dx)* command in *Func Analysis* in the catalogue, (catalogue key) .You can define the function or use the rule for the function, as shown below for $f(x) = x^2 + x$ when $x = 3$; the result is the same







Consider the linear function, $f(x) = 2x - 5$. Find the derivative of this function at several different points. What do you notice? Why? Then try some other linear functions. Explain this to a partner.

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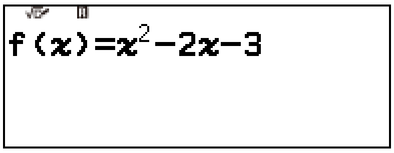
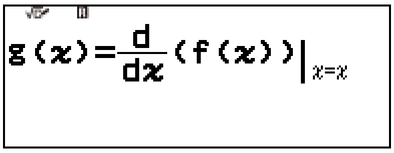
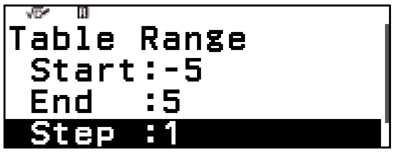
2. Consider a quadratic function, such as $f(x) = x^2 - 2x - 3$. To investigate the derivatives at several points, it's convenient to use a table. In the screens below, notice that $f(x)$ defines the value of a function at each point while $g(x)$ defines the derivative of the same function at each point.







Generate the table showing both functions. Describe and interpret the derivative values. What information do they provide about a graph of the function? Discuss this with a partner.

Change the *Table Range* settings and re-draw the table. What further information is provided?

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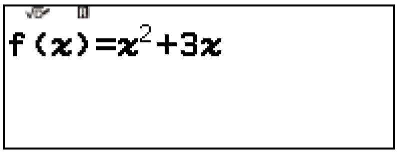
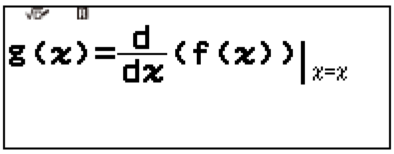
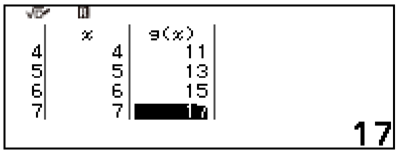
3. Examine the table below. A function and its derivative are defined, but only the derivative has been tabulated. Reproduce this situation on your *ClassWiz* and study it carefully.







What is the pattern for the values of the derivative in this case? How will it continue? Will the derivative values always be positive? Check by tapping (∨) and (∧). Discuss this with your partner.

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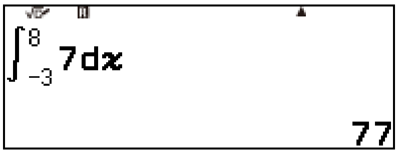
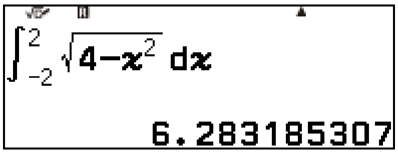
Investigating integrals

1. One way of thinking about definite integrals is that they evaluate the area under a curve. Use that idea to explain the following three results. Use your *ClassWiz* to obtain these results, using the *Integration* command in the *Func Analysis* menu in the catalogue ⊜. You will probably find it helpful to sketch the integrals involved on a separate piece of paper.





Which areas do these three integrals represent? Adjust the limits of integration or the functions – or both - to verify your thinking. Compare your observations with your partner.

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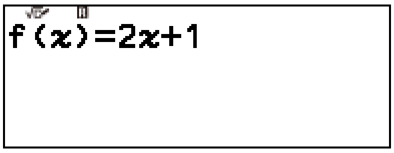
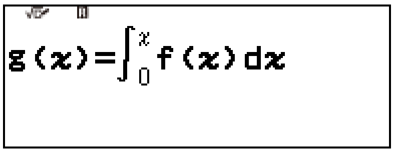
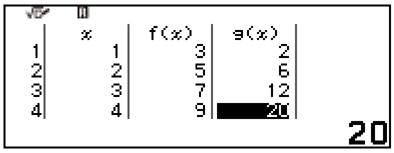
Make some more examples like these and share them with others.

2. Jacinta used a table to evaluate some definite integrals for $f(x) = 2x + 1$, as shown below. Some values for $g(x)$ are shown in the third screen below. Use your *ClassWiz* to reproduce these.







What pattern should Jacinta see in these values? Why? What will be the next terms? What happens if the lower bound is changed for the definite integral *g*? Check your answers with your partner.

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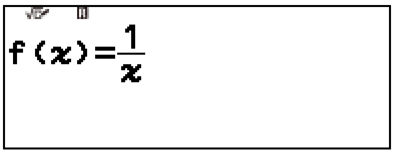
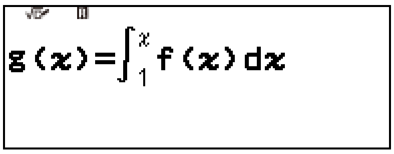
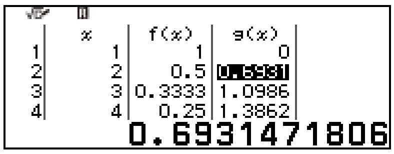
3. Yuriko studied the definite integrals below. Reproduce these on your *ClassWiz*.







Yuriko thought she recognised $\int_1^2 \frac{1}{x} dx$ as *ln* 2. Check with *ClassWiz* that she is correct.

What are the other values of the definite integral? Check several of them with your *ClassWiz* and then discuss these with your partner.

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Predict the likely value of $\int_1^e \frac{1}{x} dx$. Change an *x*-value in the table to *e* (with ⇧ ⑧) to check.

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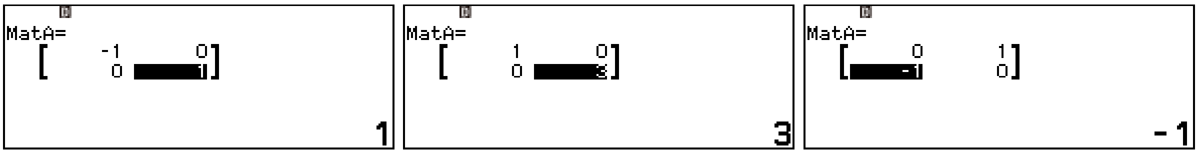
Investigating transformation matrices

1. Matrices are useful to make transformations. For example, consider a point (4,1) which can be represented in Matrix mode on *ClassWiz* by a 2 x 1 matrix **B**. The point can be transformed by pre-multiplying with a 2 x 2 matrix **A**, as shown below. The image **AB** of the transformation is obtained with *MatAMatB* via Matrix commands in ; the image point (4,-1) is shown by the screens below:

What effect does this transformation matrix **A** have on a point? Check with several other points.

Explain why the transformation matrix has this effect. Discuss this with your partner:

2. Different transformation matrices have different effects, of course. Use several different points to study the effects of each of the following three transformation matrices. It may be useful to graph the points and their images on graph paper to see what is happening.



What transformations are produced by these matrices? Explain why they have these effects.

3. Transformations of shapes can be studied by transforming their vertices. For example, use graph paper to draw the triangle XYZ with vertices X(2,1), Y(7,3) and Z(4,6). This shape can be represented as a 2 x 3 matrix **B** as shown below. Its image X'Y'Z' under the transformation matrix **A** is also shown below. Graph this image on the same axes as XYZ.

How has the triangle been transformed? What transformation matrix **A** has been used?

Use your *ClassWiz* to check your thinking. Then verify your thinking with a different triangle.

4. Investigate various transformations of your own choice on the square defined by (1,1), (5,1), (5,5) and (1,5). Predict what you expect to happen before using your *ClassWiz* to check. Share some interesting examples with your partner.

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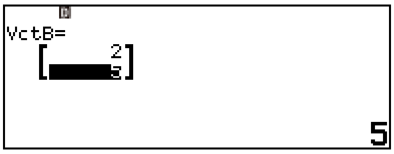
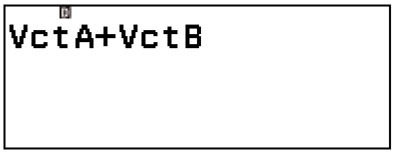
Investigating vectors

1. Consider the two vectors $\mathbf{a} = \begin{bmatrix}3\\1\end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix}2\\5\end{bmatrix}$ defined via *Tools* ⊙ in *Vector* mode on *ClassWiz*.





Use a separate piece of grid paper to draw both **a** and **b**. Use the catalogue ⊙ to find the sum **a** + **b** as shown in the third screen above. Draw the sum also on your diagram. Use your diagram to help explain how the sum is related (geometrically) to the two vectors. Check with your partner.

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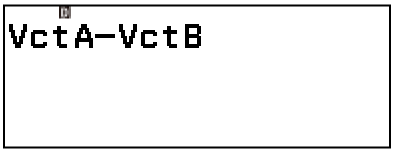
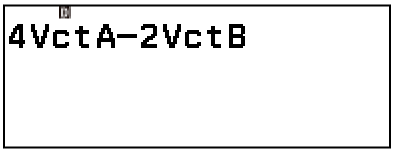
2. Using the same vectors **a** and **b** as for the previous activity, use *ClassWiz* to determine the two vectors **a** – **b** and 4**a** – 2**b**, as below. Show the first result on your diagram for the previous activity.





Check your results with your partner. Can you determine these results without using *ClassWiz*?

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3. Find the lengths of both **a** and **b** using ⊙ *Numerical Calc* and the angle between **a** and **b**, using ⊙ *Vector Calc*. Check your results with measurements on the diagram.

Now use your results to find the area of the triangle formed by **a**, **b** and **b** – **a** using the formula for the area of a triangle with sides x and y and included angle θ: Area $= \frac{1}{2}xy\sin\theta$.

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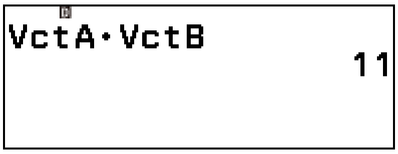
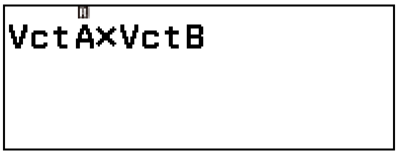
Check your results with your partner. Check also by measuring on your diagram on paper.

4. Dot products and cross products are available via ⊙ *Vector Calc* (but note that the ⊗ key can also be used for cross products). Check the two results below:





The result verifies that **a** and **b** are not orthogonal. Find a new vector **c** that is orthogonal to **a**. (Check on *ClassWiz*). Explain how the cross product **a** x **b** is related to the area found in Activity 3.

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5. Work with a partner to repeat each of Activities 1 to 4 using a different pair of vectors **a** and **b**.

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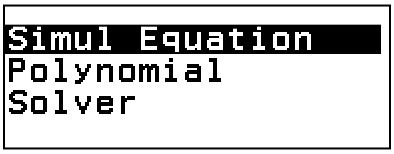
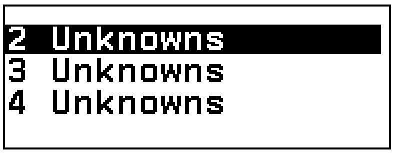
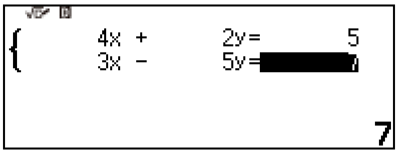
Investigating equations

1. Equations are important mathematical tools. The *Equation* mode on *ClassWiz* offers numerical solutions to various kinds of equations, as shown below. Simultaneous linear equations can be solved. Use EXE to choose this option and then to select equations with two unknowns:





Enter the equations in *x* and *y* into your *ClassWiz*, tapping EXE after each coefficient. Then tap EXE twice more to obtain the unique solution (if there is one), as in this example:

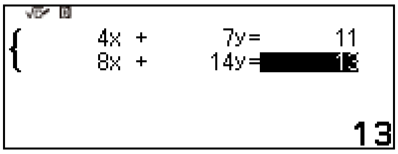


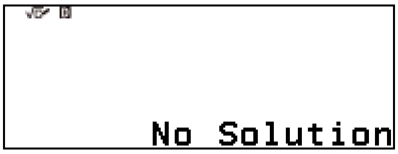
Check (mentally) that the two solutions for *x* and *y* satisfy *each* of the two equations (as the equations are to be solved *simultaneously*).

On graph paper, sketch the lines described by each equation. Where do the lines intersect? Why?

Tap EXE again and change the coefficients of the two equations. Check that the solutions are indeed solutions again. Discuss any problems with your partner.

2. Sometimes, a solution is not possible. Study this example carefully:

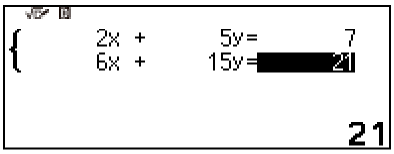




Explain why there is no solution to this system of equations. (A sketch of the lines might help.) Change the coefficients of the second equation to make another system that also has no solution.

---

3. Sometimes there is more than one solution. Study this example carefully:



Explain why this system has many solutions. (A sketch of the lines might help.) Change the coefficients of the second equation to make a different system that also has many solutions.

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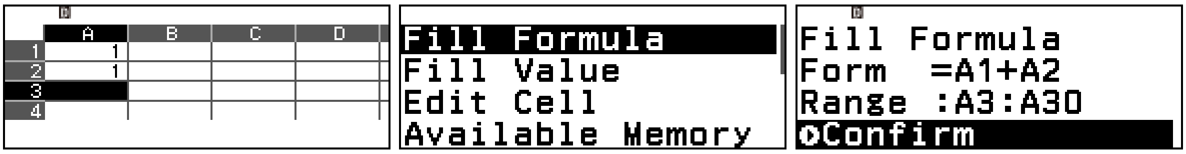
Investigating Fibonacci spreadsheets

1. The Fibonacci sequence: 1, 1, 2, 3, 5, … in which each term is the sum of the previous two terms is very famous. The sequence can be generated and studied using *Spreadsheet* mode on *ClassWiz*. Enter the first two terms in cells A1 and A2.



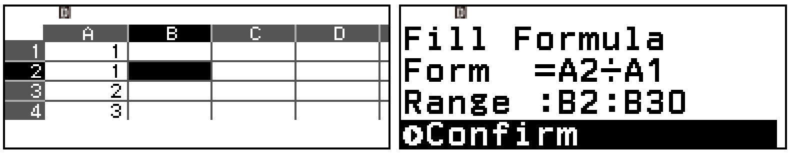
To generate more terms, highlight cell A3. Then tap (⋯) to access the *Tools* menu and select *Fill Formula*. Enter the formula for that cell (A1 + A2) using the keyboard. (Note that A is ⇧ ④). Adjust the *Range* to choose how many terms to generate, as illustrated above. Tap (EXE) to finish.

Scroll down to check that the generated terms are correct. You should find that the 14th term is 377, and is displayed as =A12+A13, consistent with the formula.

What is the 30th term of the Fibonacci Sequence?

---

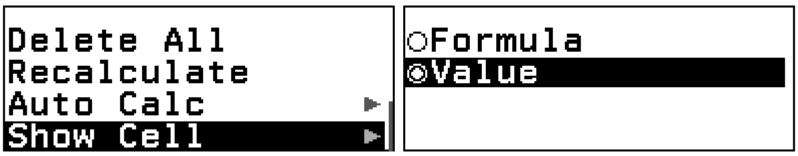
2. The Fibonacci sequence has been extensively studied. For example, the ratio of successive terms (i.e., 1÷1, 2÷1, 3÷2, 5÷3, etc …) is very interesting. To study this efficiently, use another formula, this time starting in cell B2. Enter the formula as shown and tap (EXE) to complete all the calculations.



Scroll down to check the ratios are as expected. You should find that the 6th term (on the 7th row) is 1.625, displayed as =A7÷A6. What is surprising about the later terms (from row 13 onwards)?

---

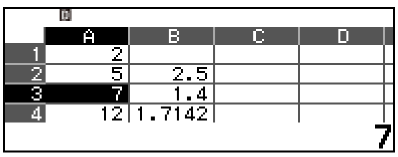
3. Because spreadsheet cells are quite small, it may not be possible to see a value completely. So you might choose to show the *value* in each cell, instead of the *formula* used to generate it. To do this use Tools (⋯) to choose *Show Cell* and select *Value* instead of *Formula*. Tap (↺) (↺) to return.



Scroll the spreadsheet to see what has changed. What do you notice? Check with a partner.

---

4. Experiment with your spreadsheet. What happens if you change (only) the first two numbers?



This is one possible change (starting with 2 and 5). Experiment with other changes like this.

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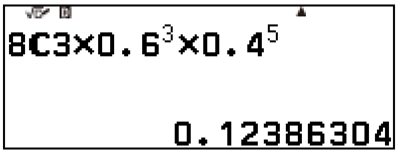
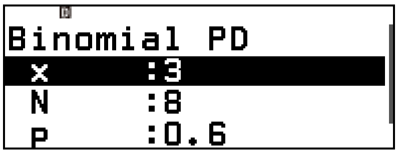
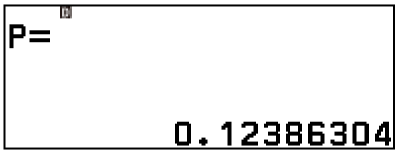
Investigating distributions

1. *The Binomial Distribution* describes theoretical probabilities, based on independent *Bernoulli* processes (i.e. a result of 0 or 1 with a constant probability, p) repeated several times. For n attempts, the probability of getting x successes is Prob $(x) = {}^{n}C_{x}\, p^{x}(1-p)^{n-x}$. For example, if a basketball player Michael is successful with free throws 60% of the time (so $p = 0.6$), the probability of getting exactly three successes in eight attempts is shown in the first screen below. (You'll need to use the *Probability* menu in ⊜ to obtain the *Combination* (C) symbol.)





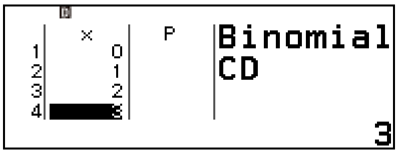


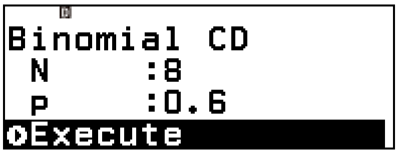
Distribution mode of *ClassWiz* allows probabilities to be obtained much more efficiently. Study the screens above to see how it is used (with *Variable* selected) to get the same result.

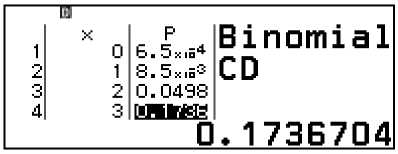
Use Binomial PD (with *List* selected) to see the probability of other numbers of successes in eight attempts. What is the most likely number of successes? Check your conclusions with a partner.

---

2. A *cumulative distribution* (CD) shows the probability that the number of wins will be less than or equal to a certain value of x. The next screens show some cumulative probabilities in the case of the free throws for Activity 1 (with *List* selected). Study them carefully and check them on *ClassWiz*:



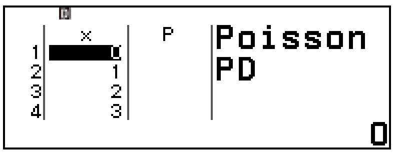


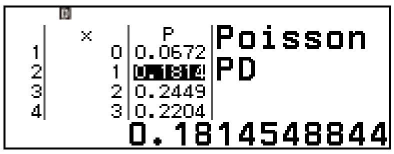


The probability is about 17% that Michael will score 3 or less in 8 throws. How likely is he to get six or more goals in 8 throws? What if he improved to be 70% successful ($p = 0.7$) each time?

Choose *Variable* instead of *List*; change the values of N and p to explore their effects.

3. The *Poisson Distribution* finds the probability of a number of discrete events occurring when their average rate of occurrence (λ) is known. In *Distribution* mode, both probabilities (PD) and cumulative probabilities (CD) can be computed. The example below shows the number of raisins likely in a cake if there are 2.7 raisins per cake on average in a cake mix.





Note that there is a probability of about 18% of a cake having only one raisin. Adjust the parameters to see how likely a cake is to have exactly four raisins. How likely is a cake to have more than five raisins? Work with a partner to check your thinking and your efficient use of *ClassWiz*.

Change the mean value (λ) to explore some other possible situations of your interest.

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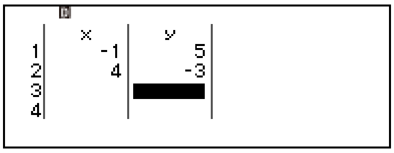
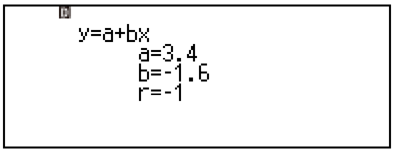
Investigating curve fitting

Bivariate data are usually analysed to detect and understand patterns and relationships, and build mathematical models using statistics, which always involves random variability. In this investigation, *ClassWiz* is used to identify relationships when no variation is involved, however, even though the *Statistics* mode is being used.

1. Consider two points in the plane, such as (-1,5) and (4,-3). The curve (in this case, a line) joining these points can be found in *Statistics* mode on *ClassWiz*. In 2-variable mode, enter the two points, and obtain the *Regression Results* for a linear model, as shown below:





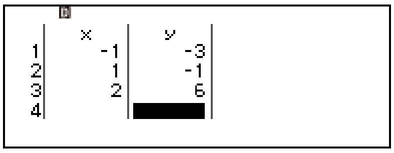
The function $y = 3.4 - 1.6x$ can be rearranged to give $8x + 5y = 17$. Check that the line is correct.

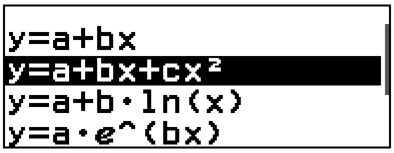
You can determine other points on the line using *Statistics Calc* and then the *Regression* commands in *Statistics* in the catalogue ⊜, as shown below (or you could find these by hand, if you wished):

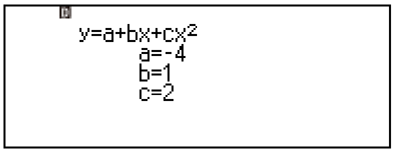
These results show that the points (2,0.2) and (2.125,0) are both on the line; check this (mentally).

Now use these processes with your own choices of two points. Check with graphs or your partner.

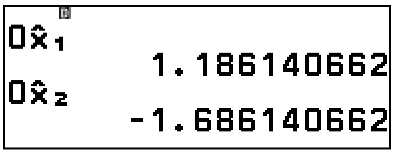
2. The same kind of idea can be used to fit a parabola to three non-collinear points, such as (-1,-3), (1,-1) and (2,6) and choosing a quadratic model, as shown below:







Read the results carefully to see that the fitted curve is $y = 2x^2 + x - 4$. Further points on the curve can be obtained as for the linear case in Activity 1, but notice that there are now two *x*-values associated with each *y*-value, unlike the linear case.



Use these processes yourself to find a quadratic curve through other sets of three points, such as (1,-3), (2,-3), (3,-5). Check in each case that the curve includes all three points.

---

3. Similar processes can be devised for fitting other curves. For example, find the exponential curve of the form $y = ab^x$ that goes through points (1,12) and (2,36). Check that the points lie on the curve.

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