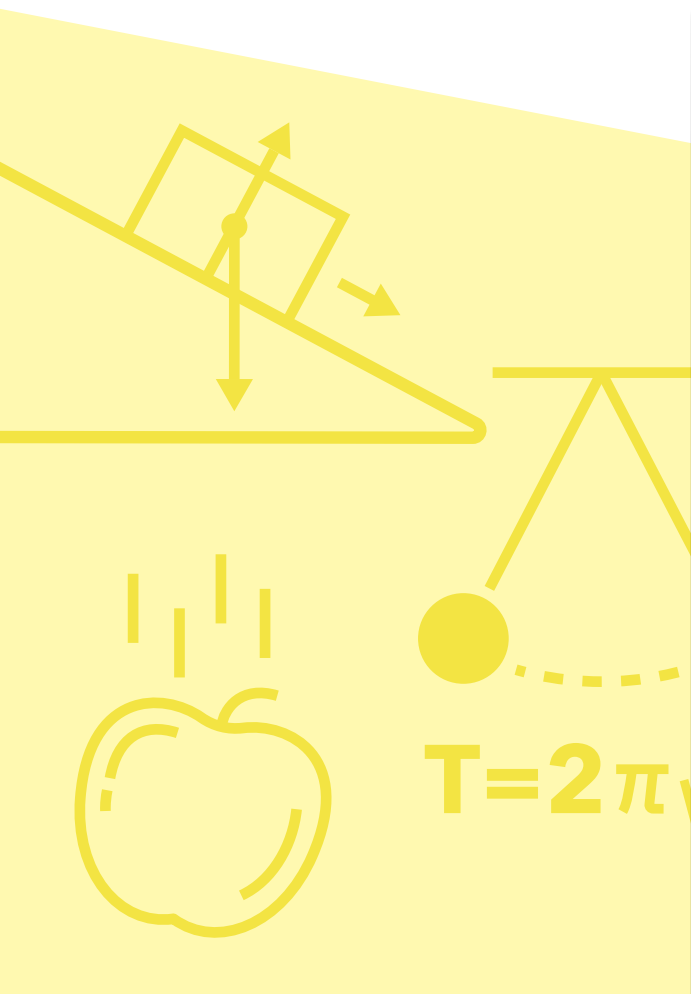
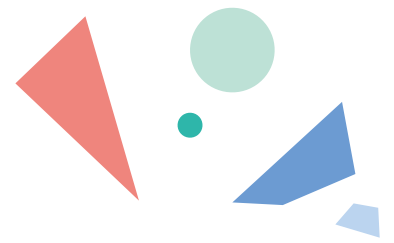


CASIO®

CASIO
TEACHING
MATERIALS

PHYSICS MATERIALS

with fx-991CW



CASIO Teaching Materials

Physics Materials with fx-991CW

Introduction

These teaching materials were created with the hope of conveying the appeal of scientific calculators to many teachers and students.

(1) Change awareness (emphasizing the thinking process) and boost efficiency in learning physics

- By reducing the time spent on manual calculations, we can have learning with a focus on the thinking process that is more efficient.
- This reduces the aversion to physics caused by complex calculations, and allows students to experience the joy of thinking, which is the essence of physics.

(2) Diversification of learning materials and problem-solving methods

- By using scientific calculators to perform calculations that are complex to do by hand, a greater variety of learning materials and problem-solving methods becomes possible.

(3) Promoting Understanding of Physical Phenomena through Data Analysis

- These materials include experimental activities in which scientific calculators are used to organize measured values and perform calculations, enabling learning that places emphasis on the analysis and interpretation of results.
- These materials and activities, which involve creating tables and graphs of experimental data using scientific calculators, foster the ability to identify patterns and relationships in physical phenomena.

Features of these teaching materials

- Makes classes more interesting by using scientific calculators
- Includes a variety of real-life problems in each unit
- Includes physics experiments in which scientific calculators are used
- Enables students to utilize the scientific calculator's functions more skillfully
- Three degrees of difficulty settings (levels 1 to 3)






**Better Physics Learning with
Scientific Calculator**

Index

- **Mechanics**
- **Waves**
- **Electromagnetism**
- **Atomic Physics**

Mechanics


Hydroelectric Power Generation Using a Dam	Level 1	1
 Estimating Height from an Object's Fall Time	Level 1	3
Train Acceleration and Hanging Angle	Level 2	5
Seesaw Balance	Level 2	6
Calculating a Car's Stopping Distance on Snow	Level 2	9
Gravitational Acceleration on Earth and the Moon	Level 2	11
Temperature effects on tank pressure and allowable pressure	Level 2	13
 Energy Conservation and Horizontal Projection	Level 2	15
 Pendulum Period	Level 2	17
Fountain Design	Level 3	19

*Level of difficulty

Level 1: Easy

Level 2: Medium

Level 3: Difficult

 : Experimental teaching materials



Hydroelectric Power Generation Using a Dam

Level 1

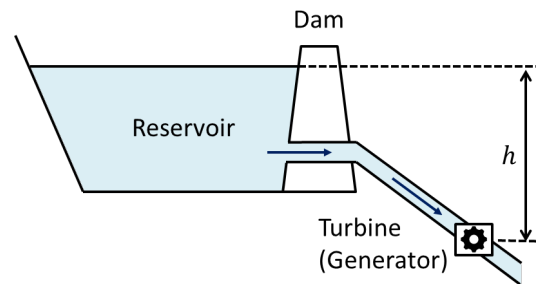
In hydroelectric power generation using a dam, the potential energy of the water in the reservoir is converted into electrical energy by turbines.



To calculate the amount of electricity generated by the dam, answer the following questions. The conditions of the dam are as follows. All answers should be given to two significant figures.

[The conditions of the dam]

- Water discharge rate: $Q = 72 \text{ m}^3/\text{s}$
- Discharge time: $t = 30 \text{ min}$
- Water density: $\rho = 1.0 \times 10^3 \text{ kg/m}^3$
- Height from turbine to lake surface: $h = 545 \text{ m}$
- Gravitational acceleration: $g = 9.8 \text{ m/s}^2$
- Power generation efficiency: 80%



Mechanism of Hydroelectric Power Generation

(1) Find the mass m (kg) of the discharged water.

- 30 min = 1800 s
- V : Volume of water released in 30 minutes

$$V[\text{m}^3] = Q[\text{m}^3/\text{s}] \times t[\text{s}]$$

$$= 72 \text{ m}^3/\text{s} \times 1800 \text{ s} = 1.296 \times 10^5 \text{ m}^3$$



To display numbers with two significant figures, change the settings as shown below.

☰	Calc Settings ▶	Input/Output ▶	○Fix ▶
	System Settings ▶	Angle Unit ▶	○Sci ▶
	Reset ▶	Number Format ▶	⊙Norm :1 ▶
	Get Started ▶	Engineer Symbol ▶	
	Sci1 : 1×10^{-1}	○Fix ▶	72x1800 1.3×10^5
	Sci2 : 1.2×10^{-1}	⊙Sci :2 ▶	
	Sci3 : 1.23×10^{-1}	○Norm ▶	
	Sci4 : 1.234×10^{-1}		

- m : Mass of water discharged in 30 minutes

$$m[\text{kg}] = \rho[\text{kg/m}^3] \times V[\text{m}^3]$$

$$= 1.0 \times 10^3 \text{ kg/m}^3 \times V[\text{m}^3] \approx 1.3 \times 10^8 \text{ kg}$$



(2) When ignoring energy losses due to inefficiency, what is the potential energy U [J] of the water used for power generation? Note that the potential energy used for power generation can be approximated by $U = mgh$.

*Strictly speaking, the water level decreases when water is discharged, so the change in h over time should be taken into account. However, since the discharge volume is negligible compared with the dam's total storage, the change in h can be ignored here.

$$\begin{aligned} U[\text{J}] &= m[\text{kg}] \times g[\text{m/s}^2] \times h[\text{m}] \\ &= m[\text{kg}] \times 9.8 \text{ m/s}^2 \times 545 \text{ m} \\ &\approx 6.9 \times 10^{11} \text{ J} \end{aligned}$$

(3) What is the actual energy E [J] considering the power generation efficiency?

$$\begin{aligned} E[\text{J}] &= 0.8 \times U[\text{J}] \\ &\approx 5.5 \times 10^{11} \text{ J} \end{aligned}$$

(4) In (3), what is the electric power P [W]?

$$\begin{aligned} P[\text{W}] &= E[\text{J}] \div t[\text{s}] \\ &= E[\text{J}] \div 1800 \text{ s} \\ &\approx 3.1 \times 10^8 \text{ W} \end{aligned}$$

(5) What is the generated energy W_e [Wh]?

$$\begin{aligned} W_e[\text{Wh}] &= P[\text{W}] \times t[\text{h}] \\ &= P[\text{W}] \times 0.5 \text{ h} \\ &\approx 1.5 \times 10^8 \text{ Wh} \end{aligned}$$

(6) If the average daily electricity consumption per household is 8.5 kWh, how many households does the generated energy W_e correspond to?

$$\begin{aligned} \cdot W_e[\text{Wh}] &= W_e \times 10^{-3} \text{ kWh} \\ \cdot W_e \times 10^{-3} \text{ kWh} \div 8.5 \text{ kWh} &\approx 1.8 \times 10^4 \end{aligned}$$

∴ It can supply 1.8×10^4 households.



Estimating Height from an Object's Fall Time

Level 1

Procedure:

A) Drop an object from your height (use any object that isn't too light, e.g., an eraser).

Measure the fall time five times and record each in the table, rounded to two decimal places.

B) Using the free-fall formula and each measured time, calculate your height and enter the results in the table.

C) Calculate and record the average fall time and average height.



A) Measured fall time (and calculated results of B and C)

	1st measurement	2nd	3rd	4th	5th	Average of 5 times C)
Fall time: $t[s]$ A)	0.62	0.55	0.57	0.61	0.63	0.60
Height: $y[m]$ B)	1.88	1.48	1.59	1.82	1.95	1.74

B) Calculation of height $y[m]$

Calculate y (height) using the free-fall formula $y = \frac{1}{2}gt^2$

- [Table], Define $f(x) = \frac{1}{2}gx^2$

- Enter the measured fall time in the table's x column and calculate the height.

Calculator interface showing the equation editor with $f(x) = \frac{1}{2}gx^2$.

Calculator interface showing a table with fall times and calculated heights. The value 0.62 is highlighted.

Calculator interface showing a table with fall times and calculated heights. The value 0.63 is highlighted.

C) Calculation of the average of five times

- [Statistics], [2-variable], Enter fall time for x and calculated height for y , then compute their averages.

Calculator interface showing the 2-Var Results screen.

Calculator interface showing the 1-Variable/2-Variable screen with $\bar{x} = 0.596$ highlighted.

Calculator interface showing a table with fall times and calculated heights. The value 0.62 is highlighted.



Consideration

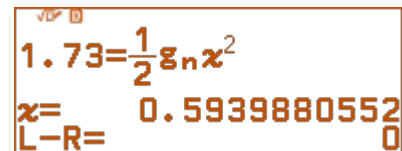
Compare the height calculated by the formula with the actual height. Think about the reasons why the values differ. Also, calculate the theoretical time it takes for an object to fall from the actual height.

- Examples of reasons why the calculated value and actual height differ
 - Timing of pressing the stopwatch (The formula includes t^2 , so differences in t lead to large errors)
 - The height differs from one's actual height
 - Air resistance
 - Variation in gravitational acceleration due to latitude
- Theoretical fall time from actual height

- When the height is 1.73 m

Calculate t (the theoretical fall time) using the free-fall formula $y = \frac{1}{2}gt^2$.

$$1.73 = \frac{1}{2}gt^2 \quad \therefore t \approx 0.59 \text{ s}$$



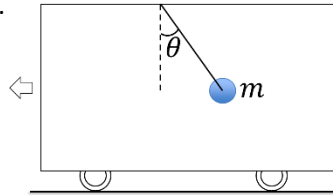


Train Acceleration and Hanging Angle

Level 2

An object of mass m hangs from the ceiling of a train.

Assume that the gravitational acceleration is $g = 9.8 \text{ m/s}^2$. Answer the following questions.



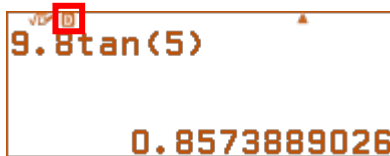
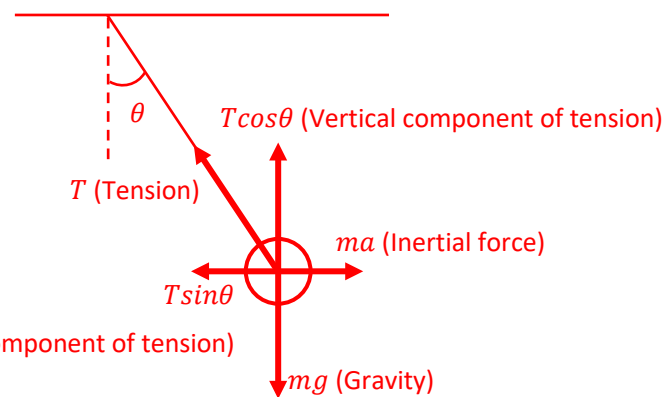
(1) When the train accelerated uniformly after departure, the object tilted at an angle of 5° as shown in the figure. Find the train's acceleration $[m/s^2]$.

Horizontal force equilibrium: $T \sin \theta = ma$ (i)

Vertical force equilibrium: $T \cos \theta = mg$ (ii)

From (i) $\frac{T \sin \theta}{T \cos \theta} = \frac{ma}{mg} \rightarrow \tan \theta = \frac{a}{g}$

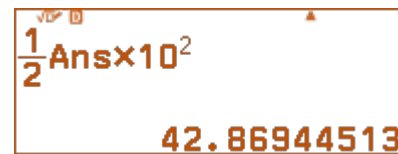
$\therefore a = g \cdot \tan \theta = 9.8 \cdot \tan 5^\circ \approx 0.86 \text{ m/s}^2$



(2) How far will the train travel if it accelerates uniformly with the value from (1) for 10 seconds after departure?

From the formula for distance in uniformly accelerated motion,

$x = \frac{1}{2} a t^2 \approx 43 \text{ m}$



(3) According to a railway company's safety standards, an acceleration of 2.0 m/s^2 is classified as "sudden acceleration" and is considered extremely dangerous. Find the angle θ for an acceleration of 2.0 m/s^2 .

From (1),

$a = g \cdot \tan \theta$

$2.0 = 9.8 \cdot \tan \theta \quad \therefore \theta \approx 12^\circ$

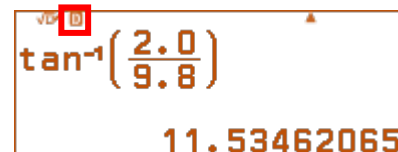


Alternative Solution (Using Inverse Trigonometric Functions)

$2.0 = 9.8 \cdot \tan \theta$

$\tan \theta = \frac{2.0}{9.8}$

$\theta = \tan^{-1} \left(\frac{2.0}{9.8} \right) \approx 12^\circ$





Seesaw Balance

Level 2

A seesaw is in balance when the weights on both sides and their distances from the fulcrum satisfy a certain relationship. Using this property, we can determine the mass of an unknown object.

For a seesaw made of a uniform board 6 m long, answer the following questions.

Seesaw I: The fulcrum is at the middle of the 6 m board.

(1) As shown in the diagram, person A stands at the position 0.5 m from the left end of the board, and person B stands at the right end, making the seesaw I balance. When person A's weight m_A is 60 kg, what is person B's weight m_B ?

Let the distance from the fulcrum to person A be l_A , the distance to person B be l_B , and the gravitational acceleration be g .

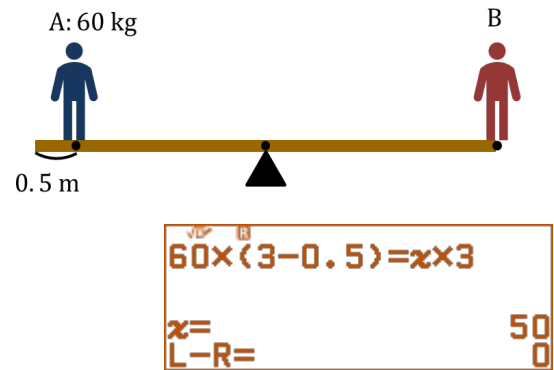
The moments about the fulcrum satisfy the following equilibrium condition:

$$m_A g \times l_A = m_B g \times l_B$$

$$m_A \times l_A = m_B \times l_B$$

$$60 \times (3 - 0.5) = m_B \times 3$$

$$m_B = 50 \text{ kg}$$



Seesaw II: The fulcrum is at 2.5 m from the left edge of the 6 m board.

(2) As shown in the figure, when person A (weight 60 kg) stands at position 1 m from the left edge of the board and person B (weight: answer from (1)) stands at position 1.95 m from the right edge of the board, the seesaw II is balanced. Find the mass of the board m .

Let the distance from the fulcrum to person A be l_A' , the distance to person B be l_B' , and the distance to the center of the board be l .

The fulcrum is located 2.5 m from the left edge of the board.

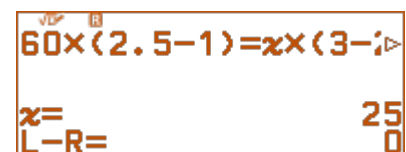
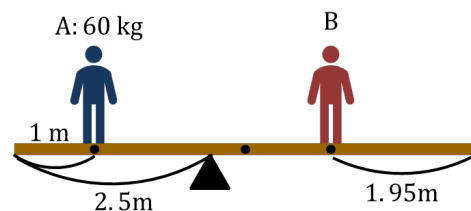
From the equilibrium of moments of force around the fulcrum,

$$m_A g \times l_A' = m g \times l + m_B g \times l_B'$$

$$m_A \times l_A' = m \times l + m_B \times l_B'$$

$$60 \times (2.5 - 1) = m \times (3 - 2.5) + 50 \times (6 - 2.5 - 1.95)$$

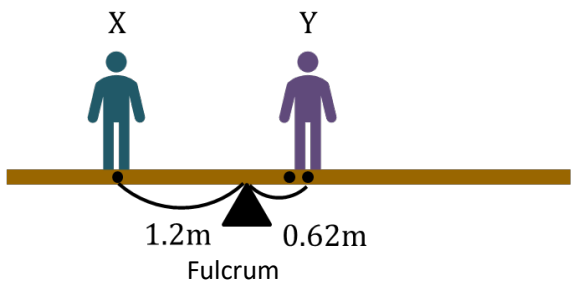
$$m = 25 \text{ kg}$$



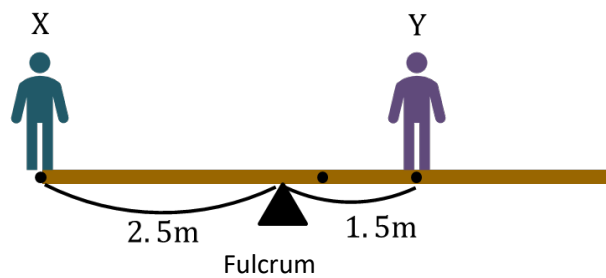
(3) When person X and person Y satisfy the following conditions, the seesaw II is balanced. Find the weights of person X (m_X) and person Y (m_Y).

	X's position	Y's position
Condition 1	1.2 m to the left of the fulcrum	0.62 m to the right of the fulcrum
Condition 2	2.5 m to the left of the fulcrum	1.5 m to the right of the fulcrum

Condition 1



Condition 2



Let the distance from the fulcrum to person X be l_X and the distance to person Y be l_Y .

Condition 1

Using the equilibrium of moments about the fulcrum, we have

$$m_X g \times l_X = m g \times l + m_Y g \times l_Y$$

$$m_X \times l_X = m \times l + m_Y \times l_Y$$

$$m_X \times 1.2 = 25 \times (3 - 2.5) + m_Y \times 0.62$$

$$1.2m_X = 12.5 + 0.62m_Y$$

$$1.2m_X - 0.62m_Y = 12.5$$

Condition 2

In the same way, the equilibrium of moments about the fulcrum gives

$$2.5m_X - 1.5m_Y = 12.5$$

Solving the system of equations

$$\begin{cases} 1.2m_X - 0.62m_Y = 12.5 \\ 2.5m_X - 1.5m_Y = 12.5 \end{cases} \quad \text{we obtain } m_X = 44 \text{ kg, } m_Y = 65 \text{ kg.}$$

$$\begin{cases} 1.2x - 0.62y = 12.5 \\ 2.5x - 1.5y = 12.5 \end{cases}$$

$$x = 44$$

$$y = 65$$

X's weight is 44 kg, and Y's weight is 65 kg.

(4) When person X and person Y from (3) ride on the seesaw II, find one additional position where the seesaw II is balanced, different from Conditions 1 and 2 in (3). Assume that X sits to the left of the fulcrum and Y sits to the right of the fulcrum. Let each distance from the fulcrum be expressed in meters to two decimal places (i.e., whole centimeters).

From the problem conditions, $0 \text{ m} \leq l_X \leq 2.5 \text{ m}$ and $0 \text{ m} \leq l_Y \leq 2.5 \text{ m}$.

$m_X \times l_X = m \times l + m_Y \times l_Y$ Therefore,

$$44 \times l_X = 25 \times 0.5 + 65 \times l_Y$$

$$65l_Y = 44l_X - 12.5$$

$$l_Y = \frac{44}{65}l_X - \frac{12.5}{65}$$

Enter the above equation into the Table, then input values for l_X from 0 to 2.5 in steps of 0.01, and find the values expressed to two decimal places.

Calculator interface showing the equation $f(x) = \frac{44x - 12.5}{65}$ and the 'Table' function selected.

Calculator interface showing the Table Range settings (Start: 0, End: 0.4, Step: 0.01) and a table of values for x and $f(x)$.

	x	$f(x)$
1	0	-0.192
2	0.01	-0.185
3	0.02	-0.178
4	0.03	-0.172

For $0 \leq l_X \leq 0.4$, there is no combination that satisfies the conditions.

Calculator interface showing the Table Range settings (Start: 0.4, End: 0.8, Step: 0.01) and a table of values for x and $f(x)$.

	x	$f(x)$
14	0.53	0.1664
15	0.54	0.1732
16	0.55	0.18
17	0.56	0.1867

For $0.4 \leq l_X \leq 0.8$, $l_X = 0.55$, $l_Y = 0.18$ meets the condition.

Answer

$$l_X = 0.55 \text{ m}, l_Y = 0.18 \text{ m} \text{ (or } l_X = 1.85 \text{ m}, l_Y = 1.06 \text{ m)}$$

✘ Since the Table can hold only up to 45 rows, the range was divided into segments such as 0 to 0.4, 0.4 to 0.8, and so on.

✘ Be aware that even if values in the Table appear to be displayed to two decimal places, moving the cursor may show additional digits.

Calculator interface showing a zoomed-in view of the table values for x and $f(x)$.

	x	$f(x)$
14	0.13	-0.104
15	0.14	-0.097
16	0.15	-0.09
17	0.16	-0.084

-0.09076923077



Calculating a Car's Stopping Distance on Snow

Level 2

On a given snow surface, a car traveling at a speed of v [km/h] began to slide as soon as the brakes were applied. The distance d [m] that the car traveled from the moment it started to slide until it came to a complete stop was measured. The results are shown in the following table.



Speed v [km/h]	36	72	90	108
Speed v' [m/s]	10	20	25	30
Distance Traveled d [m]	13	53	81	119
Coefficient of Kinetic Friction μ	0.39	0.39	0.39	0.39

Answer the following questions.

(1) Fill in the row "Speed v' [m/s]" in the table.

$$v'[\text{m/s}] = v \times \frac{1000 \text{ m}}{3600 \text{ s}} = \frac{v}{3.6} [\text{m/s}]$$

Therefore, dividing each speed value v [km/h] by 3.6 gives the corresponding values of v' [m/s].

Using a scientific calculator, you can find the values as follows.

Method 1 (Converting each value individually)

The calculator interface shows the following steps for Method 1:

- Initial display: 36
- Menu navigation: Unit Conversions
- Conversion selection: km/h to m/s
- Result for 36: 10
- Result for 72: 20
- Result for 90: 25
- Result for 108: 30

Method 2 (Converting all values at once using the Table)

Enter " $f(x) = x \text{ km/h} \rightarrow \text{m/s}$ " into the Table.

The calculator interface shows the following steps for Method 2:

- Menu navigation: Table
- Function entry: $f(x) = x \text{ km/h} \rightarrow \text{m/s}$
- Table display:

	x	$f(x)$
1	36	10
2	72	20
3	90	25
4	108	30

(2) Using the values of v' and d , fill in the row for “Coefficient of Kinetic Friction μ ” in the table. Assume that the magnitude of gravitational acceleration is 9.8 m/s^2 .

- Using μ and the normal force $N = mg$, the kinetic friction force can be expressed as $\mu N = \mu mg$. (m : mass of the car)

- From the law of conservation of energy, the kinetic energy just before braking is equal to the work done by the kinetic friction force while the car is stopping (kinetic friction force \times stopping distance d).

Therefore,

$$\frac{1}{2}mv'^2 = \mu mg \times d$$

$$\mu = \frac{v'^2}{2gd}$$

Substitute the values for v' , d , and g into this equation.

The result can be calculated as follows.

<p>Calculate Statistics Distribution Spreadsheet Table Equation</p> $\frac{10^2}{2 \times 9.8 \times 13}$ <p>0.3924646782</p>	$\frac{20^2}{2 \times 9.8 \times 53}$ <p>0.3850596843</p>
$\frac{25^2}{2 \times 9.8 \times 81}$ <p>0.3936759889</p>	$\frac{30^2}{2 \times 9.8 \times 119}$ <p>0.3858686332</p>

(3) Find the distance traveled by a car moving at 100 km/h on this snow surface from the moment it brakes until it comes to a complete stop.

From (2),

$$\mu = \frac{v'^2}{2gd}$$

$$d = \frac{v'^2}{2\mu g}$$

Substituting the value of $\mu \approx 0.39$ obtained in (2) into this equation yields

$$d \approx 101 \text{ m}$$

Therefore, the stopping distance is approximately 101 m .

$$\frac{(100 \text{ km/h} \rightarrow \text{m/s})^2}{2 \times 0.39 \times 9.8}$$

100.9425613



Gravitational Acceleration on Earth and the Moon

Level 2

Answer the following questions. Use the universal gravitational constant $G[\text{N} \cdot \text{m}^2/\text{kg}^2]$ from the scientific constants in your scientific calculator.



(1) Find the magnitude of the gravitational force F acting between two people, each weighing 50 kg, separated by a distance of 1 m. Also, calculate the force when the distance is 100 m.

$$F = G \frac{Mm}{r^2} = G[\text{N} \cdot \text{m}^2/\text{kg}^2] \times \frac{50 \text{ kg} \times 50 \text{ kg}}{1^2 \text{ m}^2} \approx 1.7 \times 10^{-7} \text{ N}$$

Calculator input: $G \times \frac{50 \times 50}{1^2}$
Result: 1.668575×10^{-7}

$$F = G \frac{Mm}{r^2} = G[\text{N} \cdot \text{m}^2/\text{kg}^2] \times \frac{50 \text{ kg} \times 50 \text{ kg}}{100^2 \text{ m}^2} \approx 1.7 \times 10^{-11} \text{ N}$$

Calculator input: $G \times \frac{50 \times 50}{100^2}$
Result: 1.668575×10^{-11}

(2) Using the fact that the gravitational force mg [N] acting on an object of mass m [kg] on Earth is approximately equal to the gravitational force between that object and Earth, calculate the value of gravitational acceleration g_e [m/s^2]. Assume Earth's radius $R_e = 6.378 \times 10^6$ m and Earth's mass $M = 5.972 \times 10^{24}$ kg.

$$mg_e = G \frac{Mm}{R_e^2}$$

$$g_e = G \frac{M}{R_e^2} = G[\text{N} \cdot \text{m}^2/\text{kg}^2] \times \frac{5.972 \times 10^{24} \text{ kg}}{(6.378 \times 10^6)^2 \text{ m}^2} \approx 9.8 \text{ N/kg} = 9.8 \text{ m/s}^2$$

Calculator input: $G \times \frac{5.972 \times 10^{24}}{(6.378 \times 10^6)^2}$
Result: 9.798429967

(3) Similarly to (2), calculate the gravitational acceleration on the Moon g_m [m/s^2]. The Moon's radius $R_m = 1.737 \times 10^6$ m, and the Moon's mass $M = 7.34 \times 10^{22}$ kg.

$$mg_m = G \frac{Mm}{R_m^2}$$

$$g_m = G \frac{M}{R_m^2} = G[\text{N} \cdot \text{m}^2/\text{kg}^2] \times \frac{7.34 \times 10^{22} \text{ kg}}{(1.737 \times 10^6)^2 \text{ m}^2} \approx 1.6 \text{ N/kg} = 1.6 \text{ m/s}^2$$

Calculator input: $G \times \frac{7.34 \times 10^{22}}{(1.737 \times 10^6)^2}$
Result: 1.623686376

(4) Express the distance an object falls freely in t seconds as y_e on Earth and y_m on the Moon. Also, find the ratio of the falling distances over the same time period.

Earth: $y_e = \frac{1}{2}g_e t^2 = \frac{1}{2} \times 9.8t^2 = 4.9t^2$

Moon: $y_m = \frac{1}{2}g_m t^2 = \frac{1}{2} \times 1.6t^2 = 0.8t^2$

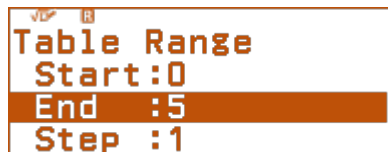


Ratio: $\frac{y_e}{y_m} = \frac{4.9t^2}{0.8t^2} = \frac{4.9}{0.8} \approx 6.1$

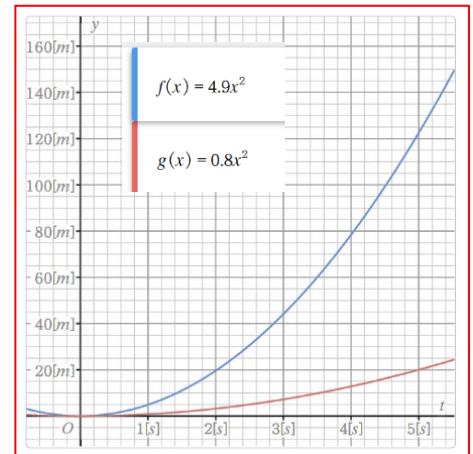
∴ The distance an object falls freely in the same time is approximately 6.1 times greater on Earth than on the Moon. (That is, on the Moon, it is approximately $\frac{1}{6}$ times that on Earth.)

(5) Plot the two equations obtained in (4) on a graph with the vertical axis: distance fallen y and the horizontal axis: time t .

Enter $y_e = f(x) = 4.9x^2$ and $y_m = g(x) = 0.8x^2$ into the [Table] function on your scientific calculator.



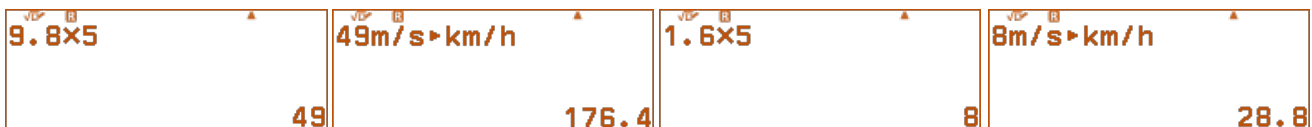
x	f(x)	g(x)
0	0	0
1	4.9	0.8
2	19.6	3.2
3	44.1	7.2



(6) Calculate the velocities of objects in free fall on Earth and the Moon after 5 s: v_e and v_m [km/h] respectively.

$v = gt$ Therefore

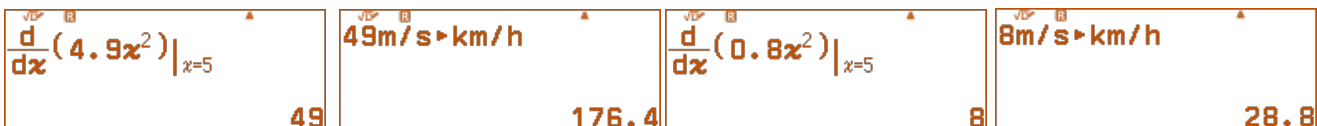
$v_e = g_e t = 9.8 \times 5 = 49 \text{ m/s} = 176.4 \text{ km/h}$ $v_m = g_m t = 1.6 \times 5 = 8 \text{ m/s} = 28.8 \text{ km/h}$



(Alternative Solution)

Find the slope of the tangent line at $t = 5$ on the graph above:

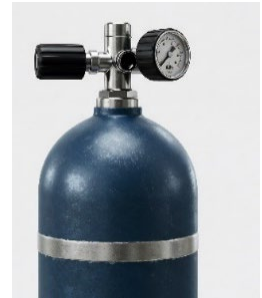
$v_e = f'(5) = 49 \text{ m/s} = 176.4 \text{ km/h}$ $v_m = g'(5) = 8 \text{ m/s} = 28.8 \text{ km/h}$





Temperature effects on tank pressure and allowable pressure Level 2

Consider an air tank containing air. The tank has a volume of 12 L, and when the temperature is 20°C, the pressure when fully filled is 200 atm. Use a scientific calculator to convert the temperature and pressure units, and apply its built-in value for the gas constant R [(Pa · m³)/(mol · K)]. Assume the gas can be treated as an ideal gas.



※Note that the unit for R is [Pa · m³/(mol · K)], which is equivalent to [J/(mol · K)].

(1) How many moles of air are required to fill the tank to 200 atm at a temperature of 20°C?

Given that the mass of 1 mol of air is 28.8 g, determine the mass of the air in grams?

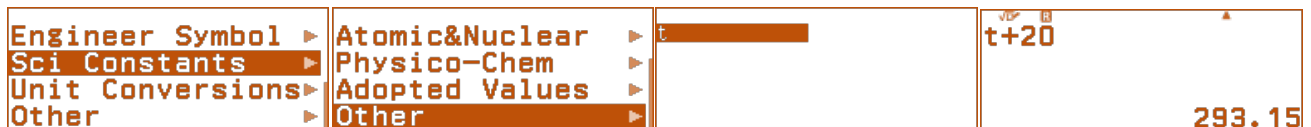
To use $PV = nRT$, convert the volume V , Temperature T , pressure P units to [m³], [K], and [Pa] respectively.

· $V = 12 \text{ L} = 12 \times 10^{-3} \text{ m}^3$

· $T = 20^\circ\text{C} = (273.15 + 20) \text{ K} = 293.15 \text{ K}$

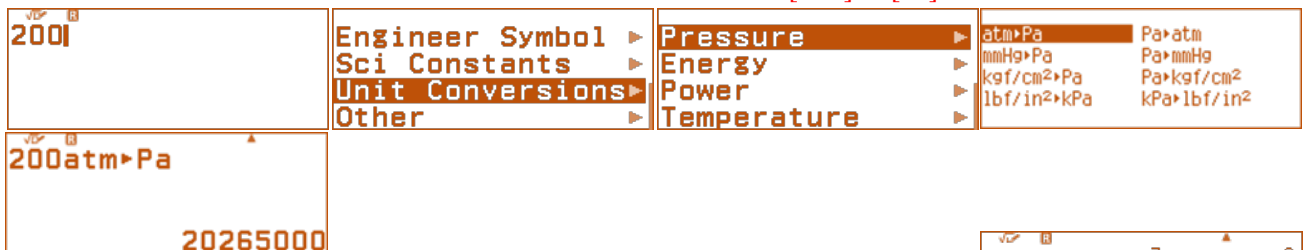
The scientific calculator has the constant $t (= 273.15)$ built in.

Add this to the Celsius temperature [°C] to get the absolute temperature [K].



· $P = 200 \text{ atm} = 2.0265 \times 10^7 \text{ Pa}$ ※1 atm = 1.01325 × 10⁵ Pa

You can use the scientific calculator's "Unit Conversions" to convert [atm] to [Pa].

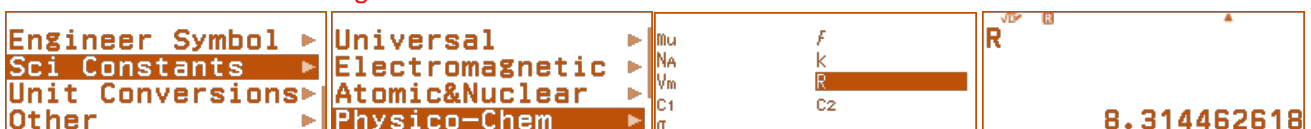


$PV = nRT, n = \frac{PV}{RT} = \frac{(2.0265 \times 10^7 [\text{Pa}]) \times (12 \times 10^{-3} [\text{m}^3])}{R[(\text{Pa} \cdot \text{m}^3)/(\text{mol} \cdot \text{K})] \times 293.15 [\text{K}]} \approx 99.77 \text{ mol} = A [\text{mol}]$

∴ The mass of the filled air is $A [\text{mol}] \times 28.8 \text{ g/mol} \approx 2873 \text{ g}$



※Use the built-in value of the gas constant R available in the scientific calculator.



(2) When a tank filled in (1) is placed in an environment of 50°C, the gas inside also reaches 50°C. To prevent the tank from rupturing under these conditions, what minimum pressure[atm] must the tank withstand?

Assume the tank's volume remains constant regardless of temperature.

$$T = 50^{\circ}\text{C} = (273.15 + 50) \text{ K} = 323.15 \text{ K}$$

$$PV = nRT$$

$$P = \frac{nRT}{V} = \frac{A[\text{mol}] \times R[(\text{Pa} \cdot \text{m}^3)/(\text{mol} \cdot \text{K})] \times 323.15 \text{ K}}{12 \times 10^{-3} \text{ m}^3}$$

$$= 22338852.98 \text{ Pa}$$

$$\approx 220 \text{ atm}$$

Calculator screenshot showing the calculation of absolute temperature: $t+50$ and the result 323.15 .

Calculator screenshot showing the calculation of pressure in Pa: $\text{AXR} \times 323.15$ over 12×10^{-3} , resulting in 22338852.98 .

Calculator screenshot showing the conversion of pressure from Pa to atm: $\text{AnsPa} \rightarrow \text{atm}$, resulting in 220.4673375 .

Alternative Solution

When the volume is constant, temperature and pressure are proportional.

Therefore, the pressure[atm] can be calculated using the absolute temperature[K] ratio.

$$T : T' = P : P'$$

$$293.15 \text{ K} : 323.15 \text{ K} = 200 \text{ atm} : x[\text{atm}]$$

Calculator screenshot showing the ratio calculation: $293.15 : 323.15 = 200 : x$, resulting in 220.4673375 .

(3) If the tank's allowable pressure is 250 atm, up to what temperature[°C] can the tank filled in (1) be used?

$$P = 250 \text{ atm} = 2.533125 \times 10^7 \text{ Pa}$$

$$PV = nRT$$

$$T = \frac{PV}{nR} = \frac{(2.533125 \times 10^7 \text{ Pa}) \times (12 \times 10^{-3} [\text{m}^3])}{A[\text{mol}] \times R[(\text{Pa} \cdot \text{m}^3)/(\text{mol} \cdot \text{K})]}$$

$$= 366.4375 \text{ K}$$

$$\approx 93^{\circ}\text{C}$$

Calculator screenshot showing the conversion of pressure from atm to Pa: $250 \text{ atm} \rightarrow \text{Pa}$, resulting in 25331250 .

Calculator screenshot showing the calculation of temperature in K: $\text{Ans} \times 12 \times 10^{-3}$ over AXR , resulting in 366.4375 .

Calculator screenshot showing the conversion of temperature from K to °C: $\text{Ans} - t$, resulting in 93.2875 .

Alternative Solution

When the volume is constant, pressure and temperature are proportional.

Therefore, the absolute temperature[K] can be calculated using the pressure[atm] ratio.

$$P : P' = T : T'$$

$$200 \text{ atm} : 250 \text{ atm} = 293.15 \text{ K} : x[\text{K}]$$

Calculator screenshot showing the ratio calculation: $200 : 250 = 293.15 : x$, resulting in 366.4375 .

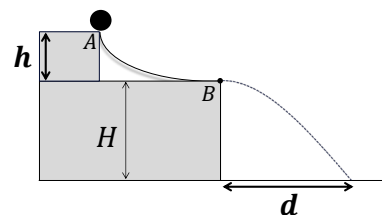


Energy Conservation and Horizontal Projection

Level 2

Procedure:

Prepare a stand as shown in the figure, where a ball of mass m rolls down the slope from point A and is projected horizontally at point B .



- 1) Fix h and H , and investigate how the horizontal distance d traveled by the ball during its fall changes when the mass m of the ball is varied.
- 2) Fix H and m , and investigate how the horizontal distance d changes when h is varied.
- 3) Explain the relationships in 1) and 2) using mathematical equations.

D) Relationship between ball mass m and horizontal distance d

mass [kg]	m_1 :	m_2 :	m_3 :	m_4 :	m_5 :
distance [m]	0.31	0.31	0.31	0.31	0.31

Fill the ball's mass measured with a balance in the table.

*The horizontal distance d is an example for $h = 0.1$ m and $H = 1$ m.

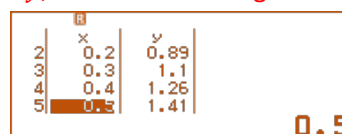
The experiment shows that d remains constant regardless of the mass.

E) Relationship between height h and horizontal distance d

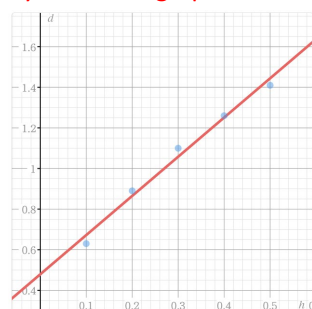
height [m]	h_1 : 0.1	h_2 : 0.2	h_3 : 0.3	h_4 : 0.4	h_5 : 0.5
distance [m]	0.63	0.89	1.10	1.26	1.41

*Example when $h = 0.1, 0.2, 0.3, 0.4,$ and 0.5 m

[Statistics], [2-variable], Enter height h as x and horizontal distance d as y , then check the regression line.



By default, a graph of the data points and the regression line is obtained.



Select the data in columns A and B, and select [Power Regression].

	A	B	C
	h	d	
1	0.1	0.63	
2	0.2	0.89	
3	0.3	1.1	
4	0.4	1.26	
5	0.5	1.41	
6			

< Back Regression

Statistics

Number

Math

abc

Handwriting

Linear Regression

MedMed Line

Quadratic Regression

Cubic Regression

Quartic Regression

Logarithmic Regression

Exponential Regression

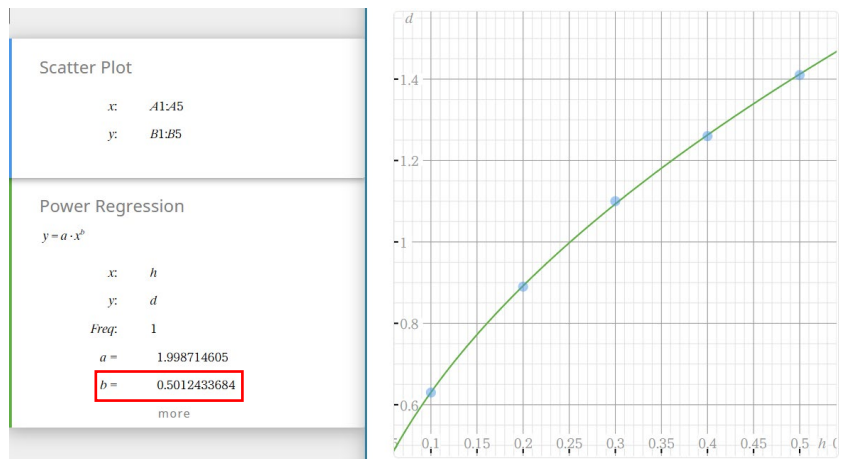
abExponential Regression

Inverse Regression

Power Regression

Sinusoidal Regression

Logistic Regression



Based on the regression analysis, the value of d is considered to be proportional to the square root of h .

F) Explain the relationships in 1) and 2) using mathematical equations.

- The horizontal distance d when the ball is projected horizontally at point B with velocity v is given by

$$d = vt \dots\dots\dots(i)$$

- The initial velocity v , based on the law of conservation of energy, is

$$\frac{1}{2}mv^2 = mgh$$

$$v = \sqrt{2gh} \dots\dots\dots(ii)$$

- The fall time t , from the equation of free fall, is

$$H = \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2H}{g}} \dots\dots\dots(iii)$$

Substituting (ii) and (iii) into (i): $d = \sqrt{2gh} \times \sqrt{\frac{2H}{g}} = 2\sqrt{hH}$

Therefore, d does not depend on m and is proportional to the square root of h .



Pendulum Period

Level 2

Check the following formula for the period of a pendulum based on the table and questions below.

$$T = 2\pi \sqrt{\frac{l}{g}}$$

T : Period[s], l : Pendulum length[m], g : Gravitational acceleration(= 9.8 m/s²)



*The equation is approximately valid when the angle of oscillation is small.

For this experiment, Keep the swing angle be 10° or less.

	Pendulum length	30 cm (= 0.30 m)						100 cm (= 1.00 m)					
		1st	2nd	3rd	4th	5th	Average	1st	2nd	3rd	4th	5th	Average
(1)	Time for 10 swings[s]	11.3	10.6	10.9	11.3	10.3	10.9	20.6	19.8	19.9	19.1	19.8	19.8
(2)	Period[s] (Measured)	1.13	1.06	1.09	1.13	1.03	1.09	2.06	1.98	1.99	1.91	1.98	1.98
(3)	Period[s] (Theoretical)	1.10						2.01					

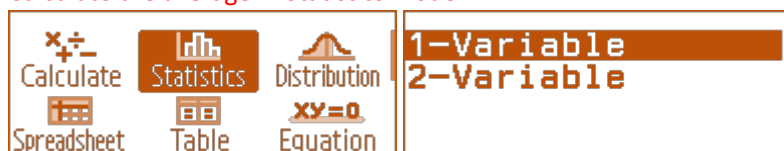
(1) Measure the time for 10 swings of a 30 cm pendulum five times and record the results in a table.

Repeat the experiment using a 100 cm pendulum and record the results in the table as well.

Also calculate the average of the five measurements for each pendulum.

Enter the experimental results in the table above. (The values above are examples.)

Calculate the average in Statistics mode.



• The Pendulum in 30 cm



• The pendulum of 100 cm



(2) From the values obtained in (1), calculate the periods of the 30 cm and 100 cm pendulums and fill them in the table.

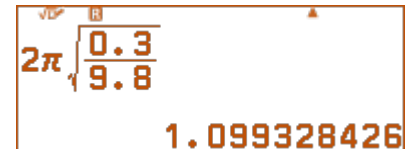
Divide each measured time in (1) by 10 to obtain the period for one swing, and enter the values in the table.

(3) Calculate the theoretical period for the 30 cm pendulum and 100 cm pendulum, and check that they are close to the experimental results.

- Theoretical value of the 30 cm pendulum's period

$$T = 2\pi \sqrt{\frac{l}{g}} = 2\pi \sqrt{\frac{0.3}{9.8}} \approx 1.10 \text{ s}$$

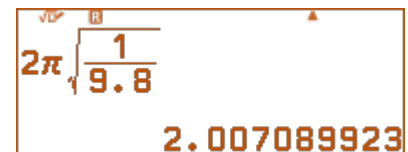
This value is almost the same as the measured period.



- Theoretical value of the 100 cm pendulum's period

$$T = 2\pi \sqrt{\frac{l}{g}} = 2\pi \sqrt{\frac{1}{9.8}} \approx 2.01 \text{ s}$$

This value is almost the same as the measured period.



(4) Make a pendulum that is as long as your height without a ruler and measure the period like (1).

Calculate your height (length of the pendulum) using the formula for the period of a pendulum

As in (1), measure the time for 10 swings, calculate the period, and use $T = 2\pi \sqrt{\frac{l}{g}}$ to find the length l .

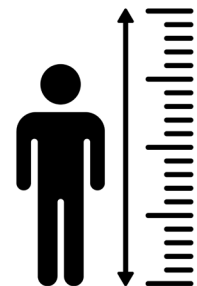
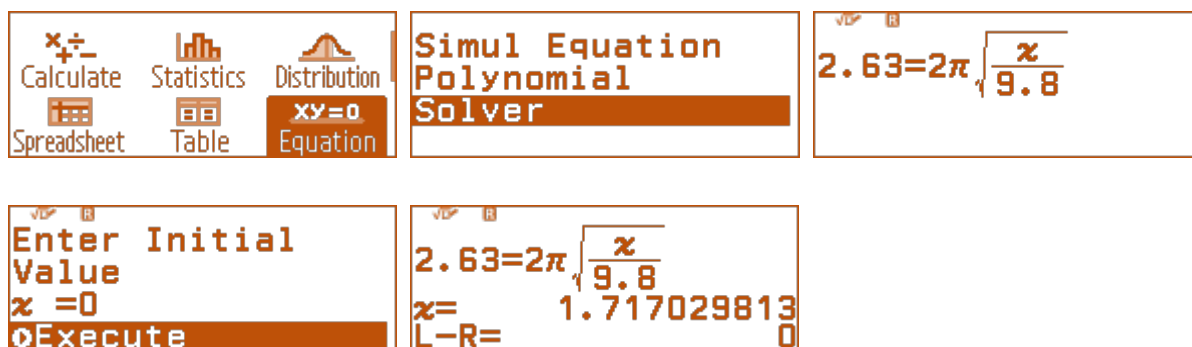
Example)

Time for 10 swings: 26.3 s

Period: $T = 26.3 \text{ s} \div 10 = 2.63 \text{ s}$

$$\therefore 2.63 = 2\pi \sqrt{\frac{l}{9.8}}$$

$l \approx 1.72 \text{ m} = 172 \text{ cm}$ So, the height is 172 cm.

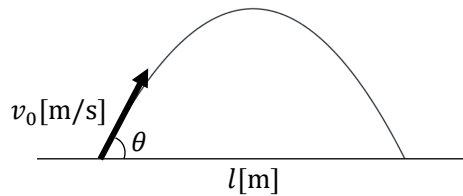





Fountain Design

Level 3

1. Consider an object launched from the ground at an angle θ [°] with initial speed v_0 [m/s], landing at a horizontal distance l [m].



(1) What is the vertical height y [m] after t [s] from launch?

Let the gravitational acceleration be g [m/s²], with upward taken as positive.

$$y = v_0 \sin \theta \cdot t - \frac{1}{2} g t^2$$

(2) What is the horizontal distance x [m] after t [s] from launch?

Let the gravitational acceleration be g [m/s²], with rightward taken as positive.

$$x = v_0 \cos \theta \cdot t$$

(3) What is the horizontal distance l [m] traveled before the object lands?

Use the identity $2 \sin \theta \cos \theta = \sin 2\theta$.

At landing, $y = 0$.

$$\text{From (1): } 0 = v_0 \sin \theta \cdot t - \frac{1}{2} g t^2$$

$$t = \frac{2v_0 \sin \theta}{g}$$

Substitute into (2):

$$l = v_0 \cos \theta \cdot t = v_0 \cos \theta \times \frac{2v_0 \sin \theta}{g} = \frac{2v_0^2 \sin \theta \cos \theta}{g} = \frac{v_0^2 \sin 2\theta}{g}$$

(4) Express the equation of the parabolic trajectory using (1) and (2). (Express y in terms of x)

Express y in terms of x by eliminating t from (1) and (2):

$$\text{From (2): } t = \frac{x}{v_0 \cos \theta}$$

Substitute into (1):

$$y = v_0 \sin \theta \cdot \frac{x}{v_0 \cos \theta} - \frac{1}{2} g \left(\frac{x}{v_0 \cos \theta} \right)^2$$

$$y = \tan \theta \cdot x - \frac{g}{2v_0^2 \cos^2 \theta} x^2$$

2. Designing a Fountain to Reach 5 m.

(1) to (4) can also be applied to the parabolic motion of a fountain.



(5) First, you fire the fountain at 45° and observe that the water lands 6 m away. Calculate the initial speed v_0 [m/s], using $g = 9.8 \text{ m/s}^2$.

From equation obtained in (3):

$$6 = \frac{v_0^2 \sin 2 \times 45^\circ}{9.8}$$

$$v_0 = \sqrt{\frac{6 \times 9.8}{\sin 90^\circ}} = \sqrt{6 \times 9.8} \approx 7.6681 \text{ m/s}$$

(6) Next, with the same initial speed in (5), find the launch angle θ [°] so that the water lands at 5 m.

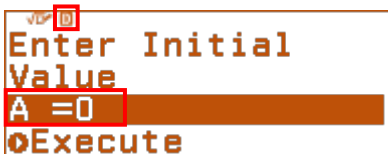
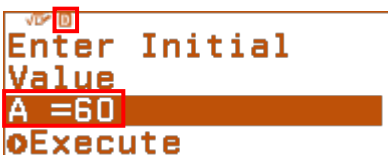
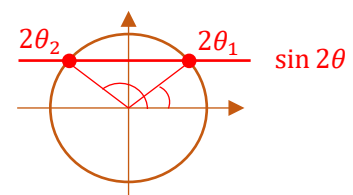
Substitute $l = 5 \text{ m}$, $v_0 = 7.6681 \text{ m/s}$ and $g = 9.8 \text{ m/s}^2$ into (3):

$$5 = \frac{7.6681^2 \times \sin 2\theta}{9.8} \quad \text{Solve for } \theta \text{ using [Solver].}$$

* Set the angle unit to Degree and use a variable (e.g. A) in place of θ .

In the range $0^\circ < \theta < 90^\circ$ ($0^\circ < 2\theta < 180^\circ$), $\sin 2\theta$ has two solutions.

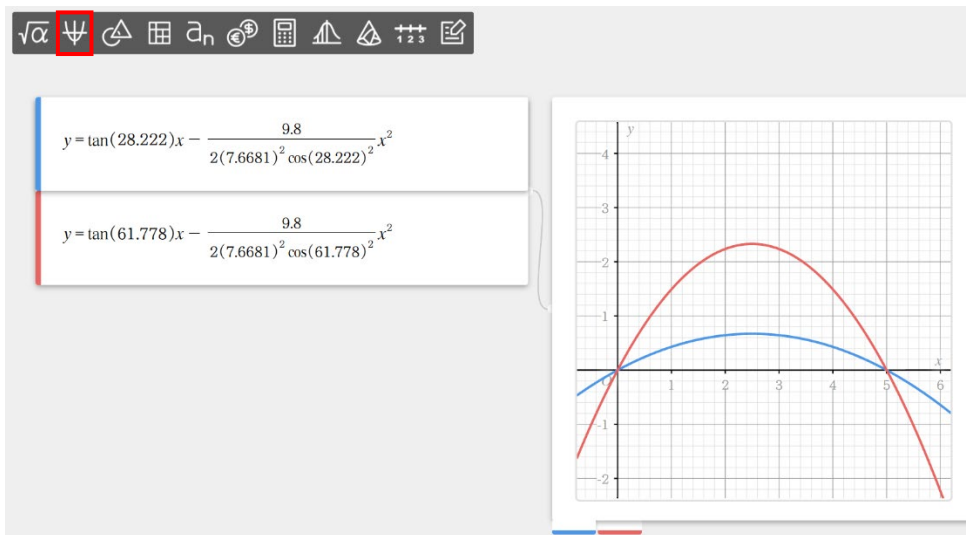
Change the initial value (for example $A=0^\circ$ and $A=60^\circ$) to find both angles.

$$\therefore \theta_1 \approx 28.222^\circ, \theta_2 = 61.778^\circ$$

(7) Using the launch conditions found in (6), draw the graph in ClassPad.net and verify that the water lands at 5 m. Substitute $v_0 = 7.6681, \theta_1 = 28.222^\circ$ (or $\theta_2 = 61.778^\circ$) into the equation obtained in (4) and draw and check the graph on ClassPad.net.

- $\theta_1 = 28.222^\circ: y = \tan\theta \cdot x - \frac{g}{2v_0^2 \cos^2 \theta} x^2 = \tan 28.222^\circ \cdot x - \frac{9.8}{2(7.6681)^2 \cos^2 28.222^\circ} x^2$
- $\theta_2 = 61.778^\circ: y = \tan\theta \cdot x - \frac{g}{2v_0^2 \cos^2 \theta} x^2 = \tan 61.778^\circ \cdot x - \frac{9.8}{2(7.6681)^2 \cos^2 61.778^\circ} x^2$



<Another way to make the graph>

Enter the two functions in [Table] and generate the graph via the QR code.

Calculate Statistics Distribution
 Spreadsheet Table Equation

$f(x) = \tan(28.222);$ $g(x) = \tan(61.778);$

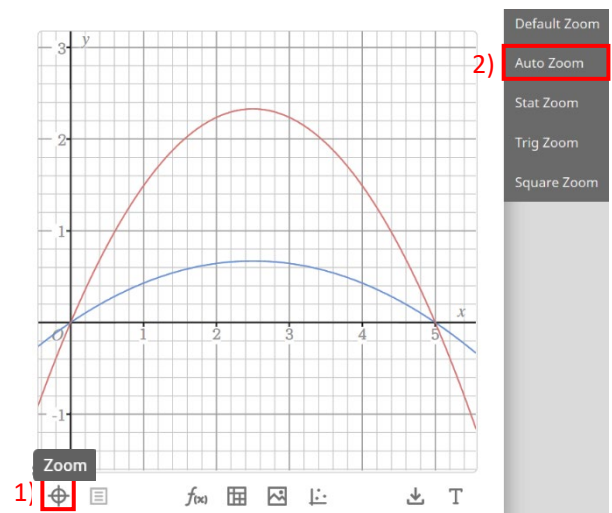
x	f(x)	g(x)
2	0.4293	1.4906
3	0.644	2.2359
4	0.644	2.2359
5	0.4293	1.4906

1


1/2

To adjust the graph range:

- 1) Click 'zoom' at the bottom left of the graph screen.
- 2) Select Auto Zoom.



Waves


Why Objects in Water Look Shallower	Level 2	1
Relationship Between Speed of Sound and Temperature	Level 2	3
 Relationship Between Sound Volume and Distance	Level 3	5

*Level of difficulty

Level 1: Easy

Level 2: Medium

Level 3: Difficult

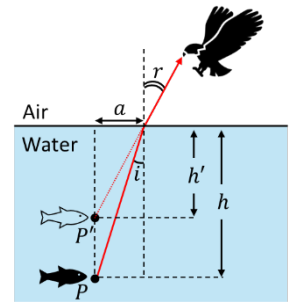
 : Experimental teaching materials



Why Objects in Water Look Shallower

Level 2

As shown in the figure, a fish is located at point P at a water depth of h . Because of the refraction of light at the water surface, the fish appears to be at point P' at the apparent depth h' . To determine the relationship between h and h' , answer the following questions.



(1) As shown in the figure, when light travels from point P into the air, write the equation for the law of refraction (Snell's law) using n_{water} , i , and r . Assume that the refractive index of air is 1 and that of water is n_{water} .

$$n_{\text{water}} \cdot \sin i = 1 \cdot \sin r$$

(2) Express $\tan i$ using h and a in the diagram. Similarly, express $\tan r$ using h' and a .

$$\tan i = \frac{a}{h} \qquad \tan r = \frac{a}{h'}$$

(3) Using a scientific calculator, verify that $\sin x \approx \tan x$ for very small values of x .

f(x) = sin(x)

x	f(x)	g(x)
1	0.0174	0.0174
2	0.0348	0.0349
3	0.0523	0.0524
4	0.0697	0.0699

1

g(x) = tan(x)

x	f(x)	g(x)
5	0.0871	0.0874
6	0.1045	0.1051
7	0.1218	0.1227
8	0.1391	0.1405

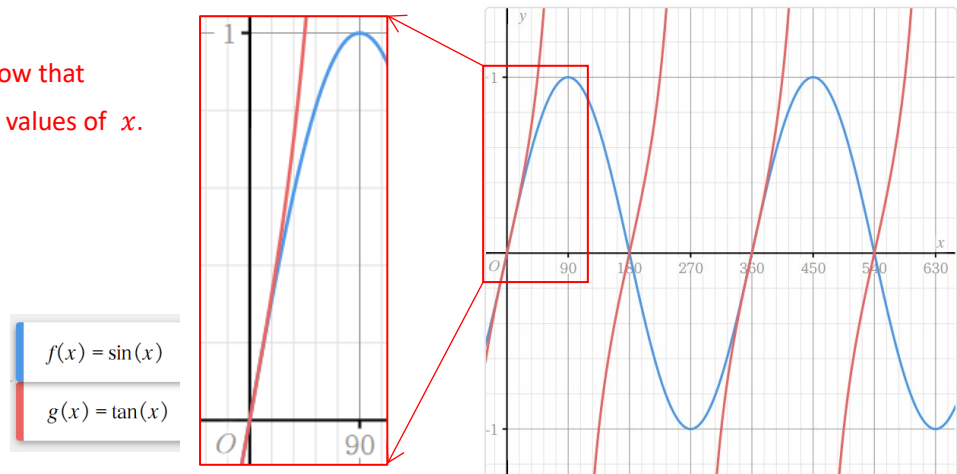
5

x	f(x)	g(x)
9	0.1564	0.1583
10	0.1736	0.1763
11	0.1908	0.1943
12	0.2079	0.2125

9

1/1

The table and the graph show that $\sin x \approx \tan x$ for very small values of x .



(4) Using the results from (1) to (3), express h' in terms of h and n_{water} assuming that i and r are very small. In this case, you may use the approximations $\sin i \approx \tan i$ and $\sin r \approx \tan r$.

Using the approximations $\sin i \approx \tan i$ and $\sin r \approx \tan r$,

$n_{water} \cdot \sin i = 1 \cdot \sin r$ in (1) can be rewritten as:

$$n_{water} \cdot \tan i = \tan r$$

Furthermore, from (2),

$$n_{water} \cdot \frac{a}{h} = \frac{a}{h'} \quad \therefore h' = \frac{h}{n_{water}}$$

(5) Using the conditions in (3), when $h = 10$ cm and $n_{water} = 1.33$, find the value of h' .

Also, using the same conditions, find the apparent depth h'' when oil with refractive index $n_{oil} = 1.47$ is used instead of water. Furthermore, compare the two and state which one appears shallower than the actual depth.

$$h' = \frac{h}{n_{water}} = \frac{10 \text{ cm}}{1.33} \approx 7.5 \text{ cm}$$

$$h'' = \frac{h}{n_{oil}} = \frac{10 \text{ cm}}{1.47} \approx 6.8 \text{ cm}$$





\therefore The apparent depth in oil is shallower than in water.

(6) When you look up at the water surface from underwater, as shown in the figure, the area outside a certain range appears completely black due to total internal reflection. This makes the water surface appear as a circular window (called Snell's window). Determine the critical angle i_c , and then find the radius b of the window visible from a depth of 30 cm ($h = 30$ cm).

From Snell's law,

$$1.33 \cdot \sin i_c = 1 \cdot \sin 90^\circ$$

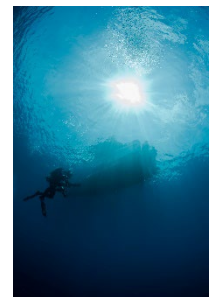
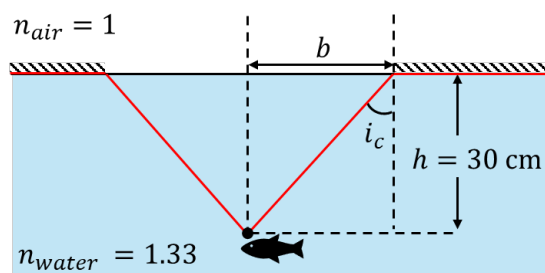
$$\sin i_c = \frac{1}{1.33}$$

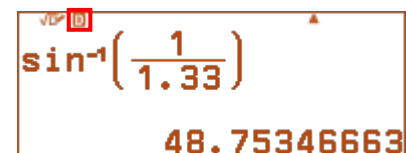
$$i_c = \sin^{-1} \frac{1}{1.33} \approx 48.8^\circ$$

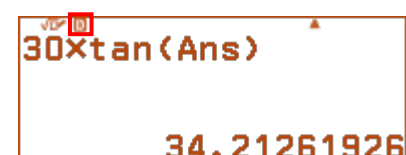
Also, from the figure

$$\tan i_c = \frac{b}{h}$$

$$b = h \cdot \tan i_c = 30 \text{ cm} \times \tan 48.8^\circ \approx 34 \text{ cm}$$









Relationship Between Speed of Sound and Temperature Level 2

Speed of sound in air: $v = 331 + 0.6T_C$ T_C : Celsius temperature [°C]

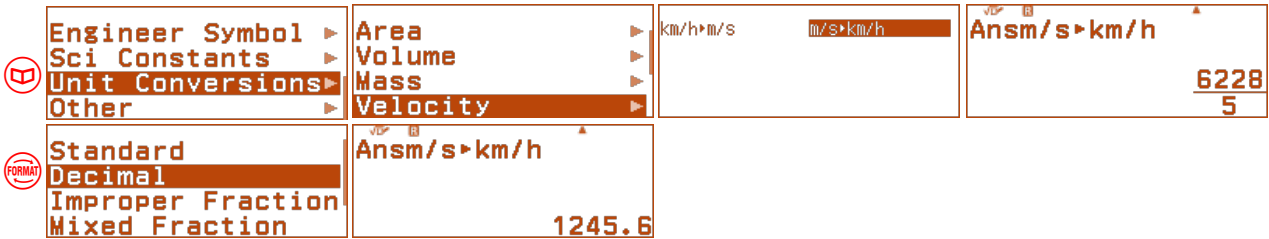


(1) Calculate the speed of sound v [m/s] at 25°C.

$$v = 331 + 0.6T_C = 346 \text{ m/s}$$



(2) Convert the speed of sound v [m/s] obtained in (1) to speed in [km/h].



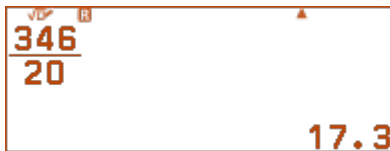
Another solution



(3) The frequency range of sound waves audible to humans is 20 Hz to 20,000 Hz.

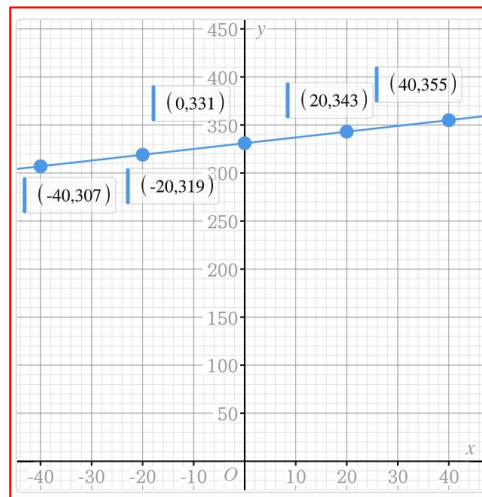
What is the approximate wavelength range of audible sound waves in air at 25°C?


Using $v = f\lambda$ for 20 Hz : $\lambda = \frac{v}{f} = \frac{346 \text{ m/s}}{20} = 17.3 \text{ m}$ for 20,000 Hz : $\lambda = \frac{v}{f} = \frac{346 \text{ m/s}}{20000 \text{ Hz}} = 0.0173 \text{ m}$



(4) Complete the following table regarding the temperature dependence of the speed of sound.

温度: T_C [°C]	音速 v [m/s]
-40	307
-30	313
-20	319
-10	325
0	331
10	337
20	343
30	349
40	355



Calculate Statistics Distribution Spreadsheet Table Equation	$f(x)$ $g(x)$ Define $f(x)$ Define $g(x)$	$f(x) = 331 + 0.6x$										
Table Range Define $f(x)/g(x)$ Table Type Edit	Table Range Start: -40 End: 40 Step: 10	<table border="1"> <thead> <tr> <th>x</th> <th>f(x)</th> </tr> </thead> <tbody> <tr><td>1</td><td>307</td></tr> <tr><td>2</td><td>313</td></tr> <tr><td>3</td><td>319</td></tr> <tr><td>4</td><td>325</td></tr> </tbody> </table>	x	f(x)	1	307	2	313	3	319	4	325
x	f(x)											
1	307											
2	313											
3	319											
4	325											
<table border="1"> <thead> <tr> <th>x</th> <th>f(x)</th> </tr> </thead> <tbody> <tr><td>5</td><td>331</td></tr> <tr><td>6</td><td>337</td></tr> <tr><td>7</td><td>343</td></tr> <tr><td>8</td><td>349</td></tr> </tbody> </table>	x	f(x)	5	331	6	337	7	343	8	349		
x	f(x)											
5	331											
6	337											
7	343											
8	349											

Another solution

$f(x)$ $g(x)$ Define $f(x)$ Define $g(x)$	Calculate Statistics Distribution Spreadsheet Table Equation	$f(x) = 331 + 0.6x$
$f(-40)$	$f(0)$	$f(20)$
307	331	343

(5) Thunder was heard 3.5 s after seeing the lightning flash. Calculate the distance from the lightning strike location to the place where the thunder was heard. Assume the air temperature is constant at 30°C.



From (4), the speed of sound at 30°C is 349 m/s.

The distance sound travels in 3.5 seconds is:

$$349 \text{ m/s} \times 3.5 \text{ s} = 1221.5 \text{ m}$$

$(331 + 0.6 \times 30) \times 3.5$
1221.5

(6) A bat uses the reflection of sound waves to determine the distance to an object. In air at 20°C a stationary bat emits a sound wave, and it takes 0.116 s for the sound wave to reflect off an object and return. Calculate the distance between the bat and the object. Also, determine how much the distance differs when the air temperature is 40°C.



- From (4), the speed of sound at 20°C is 343 m/s.

The distance traveled by the sound wave in 0.116 s is:

$$343 \text{ m/s} \times 0.116 \text{ s} = 39.788 \text{ m}$$

This is the round-trip distance between the sound source and the object, so the one-way distance is: $39.788 \div 2 = 19.894 \text{ m}$.

- Similarly, the distance at 40°C is:

$$(355 \text{ m/s} \times 0.116 \text{ s}) \div 2 = 20.59 \text{ m}$$

∴ The difference is: $20.59 \text{ m} - 19.894 \text{ m} = 0.696 \text{ m}$

$343 \times 0.116 \div 2$
19.894
$355 \times 0.116 \div 2$
20.59
Ans - 19.894
0.696



Relationship Between Sound Volume and Distance

Level 3

Procedure:

A) Measure the sound level [dB] at 1 m from a sound source using a sound level meter (or a smartphone app). Keeping the source output constant, also measure the sound level [dB] at distances of 2, 3, 4, and 5 m.



B) Using the following formula, calculate the sound intensity I at each measurement point:

$$\beta = 10 \log_{10} \frac{I}{I_0}$$

β : sound level [dB], I : sound intensity [W/m^2], I_0 : reference sound intensity (= $1.0 \times 10^{-12} \text{ W}/\text{m}^2$)

C) Investigate the relationship between the distance from the source and the corresponding sound intensity I .

A) Measured Decibels (and calculated results of B)

Distance [m]	1 m	2 m	3 m	4 m	5 m
Sound level [dB]	(1) 80	(2) 74	(3) 70.5	(4) 68	(5) 66
I [W/m^2]	(1') 1.0×10^{-4}	(2') 2.5×10^{-5}	(3') 1.1×10^{-5}	(4') 6.3×10^{-6}	(5') 4.0×10^{-6}

B) Calculation of sound intensity I [W/m^2]

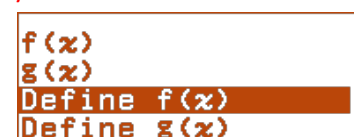
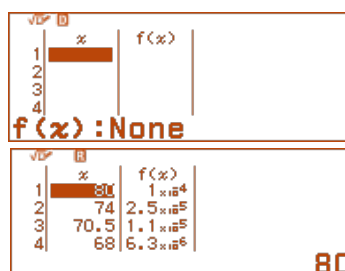
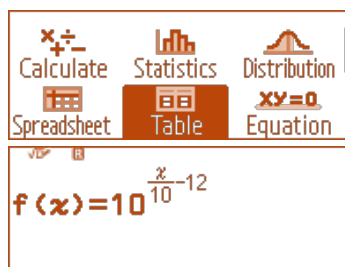
Calculate I using $\beta = 10 \log_{10} \frac{I}{I_0}$

$$\log_{10} \frac{I}{I_0} = \frac{\beta}{10}$$

$$\frac{I}{I_0} = 10^{\frac{\beta}{10}}$$

$$I = 10^{\frac{\beta}{10}} \times I_0 = 10^{\frac{\beta}{10}} \times (1.0 \times 10^{-12}) = 10^{\frac{\beta}{10}-12}$$

- [Table], Define $f(x) = I = 10^{\frac{x}{10}-12}$
- Enter the measured sound level in x column and calculate the sound intensity I .



C) The relationship between distance and sound intensity I

- [Statistics], [2-variable], Enter distance for x and sound intensity I for y , then compute their averages.

The calculator interface shows the 'Statistics' menu with '2-Variable' selected. Below it, a data table is displayed:

x	y
1	2.5×10^{-5}
2	1.1×10^{-5}
3	6.3×10^{-6}
4	4×10^{-6}
5	

- You can obtain a graph with each data point and the regression line by default.



- Select the data in columns A and B, then choose [Power Regression].

The calculator's 'Regression' menu is shown with 'Power Regression' selected. The data table from the previous step is visible on the left:

	A	B	C
	x	y	
1	1	$1E-4$	
2	2	$2.5E-5$	
3	3	$1.1E-5$	
4	4	$6.3E-6$	
5	5	$4E-6$	
6			

The calculator's regression analysis results are shown. The scatter plot displays the data points and two regression lines: a red linear regression line and a green power regression curve. The power regression equation is $y = a \cdot x^b$, with $a = 0.00009984003528$ and $b = -1.998762928$.

- Based on the regression analysis, the sound intensity I (y) is approximately inversely proportional to the square of the distance x .

Electromagnetism

Electrostatic vs. Gravitational forces: Which is stronger?	Level 1.....	1
Animal Vision and the Visible Spectrum	Level 1.....	2
Current Limits and Fuse Protection in Parallel Circuits	Level 2.....	4
Power Loss in Wires	Level 3.....	6

*Level of difficulty

Level 1: Easy

Level 2: Medium

Level 3: Difficult



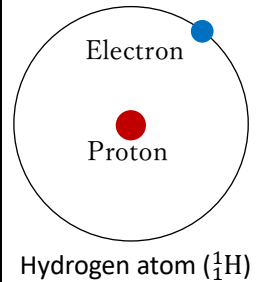
: Experimental teaching materials



Electrostatic vs. Gravitational forces: Which is stronger? Level 1

The motion of planets is controlled by gravity, and in daily life, we experience the effects of gravity here on Earth. On the other hand, electrostatic force is often considered weak, but is that actually true? Calculate and compare the gravitational and electrostatic forces between a proton and an electron in a hydrogen atom using the following data.

Constants	Symbols on fx-991CW	Value [Unit]
proton mass	m_p	1.67×10^{-27} kg
electron mass	m_e	9.11×10^{-31} kg
gravitational constant	G	6.67×10^{-11} N · m ² /kg ²
hydrogen atom radius (Bohr radius)	a_0	5.29×10^{-11} m
elementary charge	e	1.60×10^{-19} C
Coulomb constant*	-	8.99×10^9 N · m ² /C ²



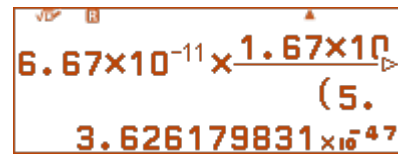
*Since the Coulomb constant $k = \frac{1}{4\pi\epsilon_0}$, using the scientific constant (ϵ_0) in calculator gives a more accurate value.

(1) Calculate the gravitational force F_1 between the proton and electron in a hydrogen atom.

$$F_1 = G \frac{m_p m_e}{a_0^2} = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \times \frac{1.67 \times 10^{-27} \text{ kg} \times 9.11 \times 10^{-31} \text{ kg}}{(5.29 \times 10^{-11} \text{ m})^2} \approx 3.6 \times 10^{-47} \text{ N}$$



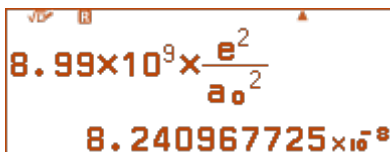
Using scientific constants on your scientific calculator



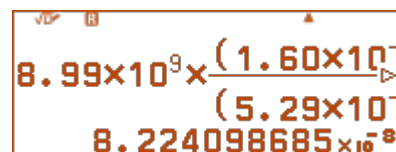
Entering the values manually

(2) Calculate the electrostatic force F_2 between the proton and electron in a hydrogen atom.

$$F_2 = k \frac{e^2}{a_0^2} = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \times \frac{(1.60 \times 10^{-19} \text{ C})^2}{(5.29 \times 10^{-11} \text{ m})^2} \approx 8.2 \times 10^{-8} \text{ N}$$



Using scientific constants on your scientific calculator

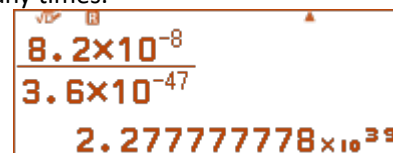


Entering the values manually

(3) Based on (1) and (2), determine which force is larger and by how many times.

$$\text{From (1) and (2), } \frac{F_2}{F_1} = \frac{8.2 \times 10^{-8}}{3.6 \times 10^{-47}} \approx 2.3 \times 10^{39}$$

The electrostatic force is 2.3×10^{39} times larger.



✧ From a macroscopic perspective, the positive charge of protons and the negative charge of electrons in matter are equal, resulting in electrical neutrality. This makes the gravitational force dominant at macroscopic scales.

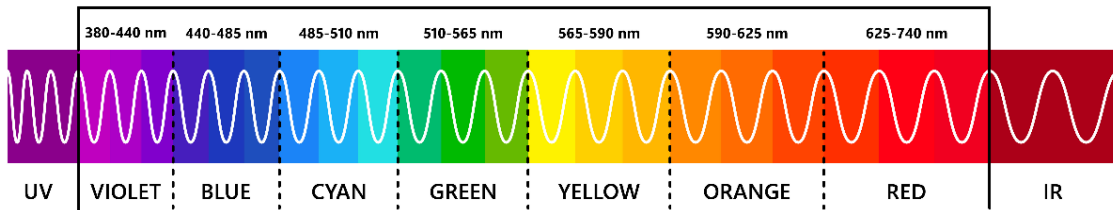


Animal Vision and the Visible Spectrum

Level 1

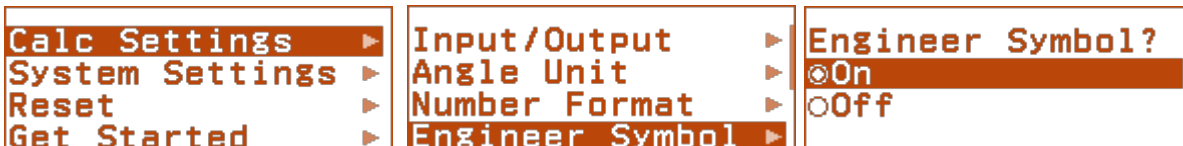
The range of light wavelengths that animals can see varies by species. To explore these differences, answer the following questions.

(1) Calculate the wavelength of light with frequencies of 9.1×10^{14} Hz, 7.4×10^{14} Hz, 5.8×10^{14} Hz, 5.1×10^{14} Hz, 4.6×10^{14} Hz, 3.7×10^{14} Hz. Also state the color each would appear to the human eye. Refer to the figure below and summarize your answers in the table. If the light is outside the visible spectrum, write “UV” or “IR.”



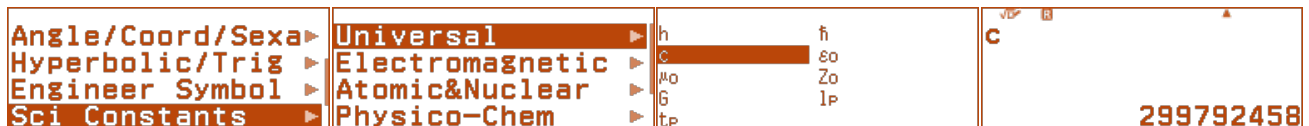
	Frequency [Hz]	9.1×10^{14}	7.4×10^{14}	5.8×10^{14}	5.1×10^{14}	4.6×10^{14}	3.7×10^{14}
(1)	Wavelength [nm]	329	405	517	588	652	810
	Color	UV	VIOLET	GREEN	YELLOW	RED	IR
(2)	Human	X	✓	✓	✓	✓	X
	Honeybee	✓	✓	✓	✓	X	X
	Hummingbird	✓	✓	✓	✓	✓	X

To display the value in nanometers, press Ⓜ , select “Engineer Symbol” from “Calc Settings”, and set it to “On”.



*Use the value of the speed of light c [m/s] from the scientific constants in the scientific calculator.

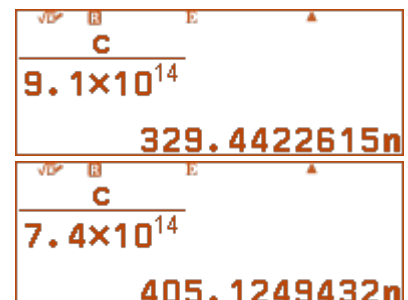
Press Ⓜ and make the following selections.



$$c = f\lambda$$

$$\lambda = \frac{c}{f} = \frac{3.0 \times 10^8 \text{ m/s}}{9.1 \times 10^{14} \text{ Hz}} \approx 329 \text{ nm} \quad \therefore \text{UV}$$

$$\lambda = \frac{c}{f} = \frac{3.0 \times 10^8 \text{ m/s}}{7.4 \times 10^{14} \text{ Hz}} \approx 405 \text{ nm} \quad \therefore \text{VIOLET}$$



$$\lambda = \frac{c}{f} = \frac{3.0 \times 10^8 \text{ m/s}}{5.8 \times 10^{14} \text{ Hz}} \approx 517 \text{ nm} \quad \therefore \text{GREEN}$$

$$\lambda = \frac{c}{f} = \frac{3.0 \times 10^8 \text{ m/s}}{5.1 \times 10^{14} \text{ Hz}} \approx 588 \text{ nm} \quad \therefore \text{YELLOW}$$

$$\lambda = \frac{c}{f} = \frac{3.0 \times 10^8 \text{ m/s}}{4.6 \times 10^{14} \text{ Hz}} \approx 652 \text{ nm} \quad \therefore \text{RED}$$

$$\lambda = \frac{c}{f} = \frac{3.0 \times 10^8 \text{ m/s}}{3.7 \times 10^{14} \text{ Hz}} \approx 810 \text{ nm} \quad \therefore \text{IR}$$

$\frac{c}{f}$	$\frac{3.0 \times 10^8 \text{ m/s}}{5.8 \times 10^{14} \text{ Hz}}$	$\approx 516.8835483 \text{ nm}$
$\frac{c}{f}$	$\frac{3.0 \times 10^8 \text{ m/s}}{5.1 \times 10^{14} \text{ Hz}}$	$\approx 587.828349 \text{ nm}$
$\frac{c}{f}$	$\frac{3.0 \times 10^8 \text{ m/s}}{4.6 \times 10^{14} \text{ Hz}}$	$\approx 651.7227348 \text{ nm}$
$\frac{c}{f}$	$\frac{3.0 \times 10^8 \text{ m/s}}{3.7 \times 10^{14} \text{ Hz}}$	$\approx 810.2498865 \text{ nm}$

(2) Given the visible spectrum of humans, honeybees, and hummingbirds shown below, determine whether each can see the light described in (1). If they can see it, mark \checkmark ; if they cannot, mark X in the table in (1).

Visible spectrum — Humans: 380–740 nm, Honeybees: 300–650 nm, Hummingbirds: 300–700 nm

Enter your answers in Table (1)

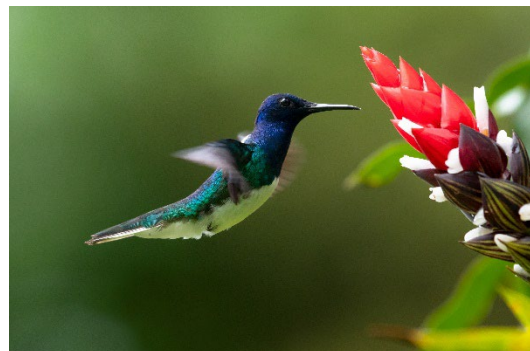
(3) Which flower color is likely to be least common among flowers pollinated by honeybees? Also, explain your reasoning.

Sample Answer: It is likely that there are few red flowers.

Reason: For honeybees, red light lies outside the visible spectrum and is therefore invisible.

(4) It is well known that hummingbirds often visit red flowers to sip nectar. Explain why.

Sample Answer: Hummingbirds can perceive longer-wavelength light (red) than honeybees. Furthermore, since honeybees have difficulty detecting red flowers and are less likely to gather there, it is thought that hummingbirds frequently visit red flowers because they have fewer competitors.

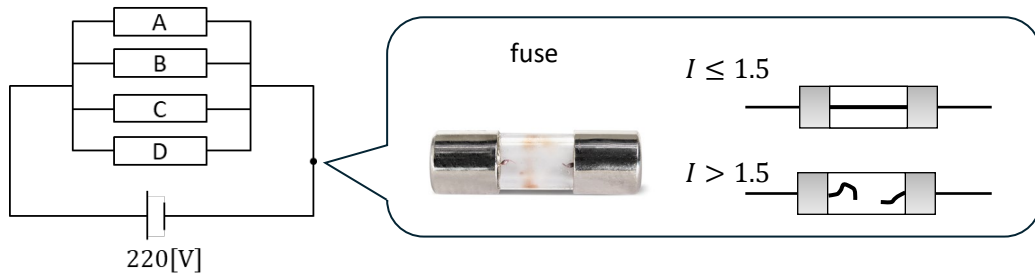




Current Limits and Fuse Protection in Parallel Circuits

Level 2

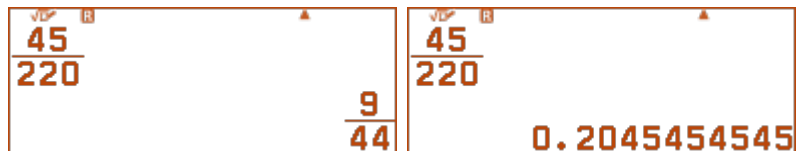
Electronic devices A, B, C, and D are connected in parallel to a 220 V DC power supply, as shown in the diagram. Each device is treated as an ideal resistor, and their power consumption is A: 45 W, B: 60 W, C: 85 W, D: 90 W. To prevent overcurrent, the circuit is designed so that if the total current through the entire circuit exceeds 1.5 A, the fuse will blow and stop the current.



(1) Calculate the current through device A when only A is used. (Assume no current flows through B, C, and D.)

The voltage across A is 220 V. Using the power formula $P = IV$,

$$I_A = \frac{P_A}{V_A} = \frac{45 \text{ W}}{220 \text{ V}} = \frac{9}{44} \approx 0.20 \text{ A}$$



(2) When A, B, C, and D are used at the same time, determine the current through each device. Also, calculate the total current in the circuit and determine whether the fuse will blow.

Since it is a parallel circuit, each device has a voltage of 220 V. $P = IV$ Therefore,

$$A: I_A = \frac{P_A}{V_A} = \frac{45 \text{ W}}{220 \text{ V}} = \frac{9}{44} \approx 0.20 \text{ A}$$

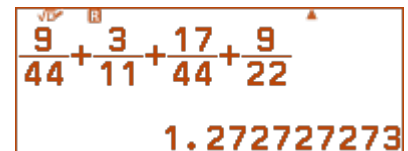
$$B: I_B = \frac{P_B}{V_B} = \frac{60 \text{ W}}{220 \text{ V}} = \frac{3}{11} \approx 0.27 \text{ A}$$

$$C: I_C = \frac{P_C}{V_C} = \frac{85 \text{ W}}{220 \text{ V}} = \frac{17}{44} \approx 0.39 \text{ A}$$

$$D: I_D = \frac{P_D}{V_D} = \frac{90 \text{ W}}{220 \text{ V}} = \frac{9}{22} \approx 0.41 \text{ A}$$

The total current in the entire circuit is the sum of the currents through each branch.

$$I_{total} = I_A + I_B + I_C + I_D = \frac{9}{44} + \frac{3}{11} + \frac{17}{44} + \frac{9}{22} = \frac{14}{11} \approx 1.27 \text{ A}$$



∴ The total current in the circuit is 1.5 A or less, so the fuse will not blow.

Alternative Solution (Using the Table to calculate each current)

Let x represent the power consumption of each device, and let $f(x)$ represent the corresponding current.



$$f(x) = \frac{x}{220}$$

	x	f(x)
1	45	0.2045
2	60	0.2727
3	85	0.3863
4	90	0.409

$$I_{total} = I_A + I_B + I_C + I_D = \frac{14}{11} \approx 1.27 \text{ A}$$

$$f(45) + f(60) + f(85) = 1.272727273$$

(3) You want to connect an additional device E in parallel, just like devices A, B, C, and D, and use all of them at the same time. To prevent the fuse from blowing, what is the maximum power consumption [W] of device E?

When device E is connected in parallel, the maximum current that can flow through E is

$$I_E = 1.5 - (I_A + I_B + I_C + I_D) = \frac{5}{22} \approx 0.23 \text{ A}$$

$$1.5 - \text{Ans} = 0.2272727273$$

※The value for Ans on this screen is the value for I_{total} in (2).

Since device E is connected in parallel, the voltage across E is also 220 V.

Therefore, the power consumption of device E is

$$P_E = I_E V_E = \frac{5}{22} \times 220 = 50 \text{ W}$$

$$\text{Ans} \times 220 = 50$$

※The answer on this screen is the value of I_E

∴ The maximum additional power consumption of device E that can be used is 50 W.

Verification

Use the fact that "total circuit power consumption" equals "the sum of the power consumed in each branch".

- Total circuit power consumption: $P = IV = 1.5 \text{ A} \times 220 \text{ V} = 330 \text{ W}$

$$1.5 \times 220 = 330$$

- Total power consumption of devices A, B, C, D and E: $45 \text{ W} + 60 \text{ W} + 85 \text{ W} + 90 \text{ W} + 50 \text{ W} = 330 \text{ W}$

$$45 + 60 + 85 + 90 + 50 = 330$$

Therefore, we can confirm that the two are equal.



Power Loss in Wires

Level 3

Because wires have resistance, some power is lost during transmission. The power loss $P_{loss}[W]$ in wires depends on the current $I[A]$ and the wire resistance $R[\Omega]$ as shown below.

$$P_{loss} = I^2 R$$

To examine P_{loss} , answer the following questions.



(1) Wires with a length $l = 3.0 \times 10^4 \text{ m}$ and cross-sectional area $A = 2.5 \times 10^{-4} \text{ m}^2$ were made from the following materials. Find the resistance R and the power loss P_{loss} when a current of 5 A flows through each wire. Fill in the table below.

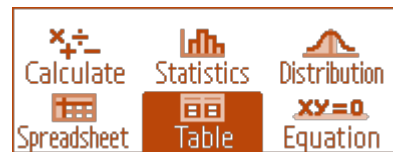
Material	Resistivity $\rho[\Omega \cdot \text{m}]$	Resistance $R[\Omega]$	Power Loss $P_{loss}[W]$
Copper	1.7×10^{-8}	2.04	51
Aluminum	2.7×10^{-8}	3.24	81
Nichrome	1.1×10^{-6}	132	3300

The resistance of a wire can be calculated using the formula on the right. $R = \rho \times \frac{l}{A}$

By entering the formulas below into the "Table" of a scientific calculator, you can calculate both R and P_{loss} .

$$f(x): R = \rho \times \frac{l}{A} = x \times \frac{3.0 \times 10^4}{2.5 \times 10^{-4}} \quad * \text{ Here, } \rho \text{ is replaced with } x.$$

$$g(x): P_{loss} = I^2 R = 5^2 \times f(x) \quad * \text{ Here, } R \text{ is replaced with } f(x).$$



$$f(x) = x \times \frac{3.0 \times 10^4}{2.5 \times 10^{-4}}$$

$$g(x) = 5^2 \times f(x)$$

x	f(x)	g(x)
1	1.7×10^8	2.04
2	2.7×10^8	3.24
3	1.1×10^6	132
4		

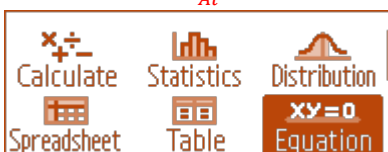
By entering each value of ρ for x , you can calculate the values of $f(x)$ and $g(x)$ (R and P_{loss}).

(2) Aluminum (Al) is cheaper than copper (Cu). Therefore, we want to make an aluminum wire with the same length and resistance as the copper wire in (1). What is the required cross-sectional area A ?

$$R_{Al} = R_{Cu}$$

$$\rho_{Al} \times \frac{l}{A_{Al}} = \rho_{Cu} \times \frac{l}{A_{Cu}}$$

$$2.7 \times 10^{-8} \times \frac{3.0 \times 10^4}{A_{Al}} = 1.7 \times 10^{-8} \times \frac{3.0 \times 10^4}{2.5 \times 10^{-4}} \quad \therefore A_{Al} \approx 4.0 \times 10^{-4} \text{ m}^2$$



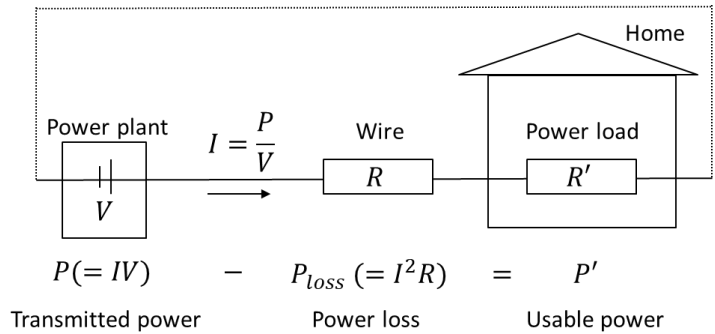
Simul Equation Polynomial Solver

$$2.7 \times 10^{-8} \times \frac{3.0 \times 10^4}{A} =$$

$$A = 3.9705882 \times 10^{-4}$$

$$L-R = 0$$

(3) As shown in the figure, assume that power $P = 1.5 \times 10^4 \text{ W}$ is transmitted from the power plant at the voltages listed in the table below. For each case, calculate the current $I[\text{A}]$ and the power loss $P_{\text{loss}}[\text{W}]$ in the wire, and fill in the table. The total resistance of the wire is $R = 3.0 \Omega$.



Voltage: $V[\text{V}]$	(3) Current: $I[\text{A}]$	(3) Power Loss: $P_{\text{loss}}[\text{W}]$	(5) Power Loss Rate: $\frac{P_{\text{loss}}}{P} \times 100[\%]$
1.0×10^3	15	675	4.5
1.0×10^4	1.5	6.75	0.045
1.0×10^5	0.15	0.0675	0.00045

The value of the current can be found from $P = IV$ as $I = \frac{P}{V}$.

By entering the formula below into the “Table” of a scientific calculator, you can calculate both I and P_{loss} .

$f(x): I = \frac{P}{V} = \frac{1.5 \times 10^4}{x}$ * Here, V is replaced with x .

$g(x): P_{\text{loss}} = I^2 R = f(x)^2 \times 3.0$ * Here, I is replaced with $f(x)$.

x	$f(x)$	$g(x)$
1000	15	675
10000	1.5	6.75
100000	0.15	0.0675

By inputting each value of V for x , you can calculate the values of $f(x)$ and $g(x)$ (I and P_{loss}).

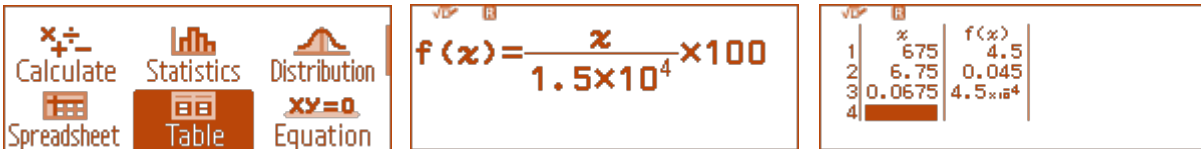
(4) In actual power transmission, electricity is transmitted at high voltage before it is stepped down near each household. Explain why, based on the result from (3).

When transmitting power P , increasing the voltage V reduces the current $I = \frac{P}{V}$. This decreases the power loss in the wires ($P_{\text{loss}} = I^2 R$).

(5) Calculate the ratio of P_{loss} to the transmitted power $P = 1.5 \times 10^4$ W (the power loss rate[%]) and fill in the table.

Using the “Table” on a scientific calculator, enter the formulas below to calculate the values at the same time.

$$f(x): \text{power loss rate}[\%] = \frac{P_{loss}}{P} \times 100 = \frac{x}{1.5 \times 10^4} \times 100 \quad * \text{ Here, } P_{loss} \text{ is replaced with } x.$$



	x	f(x)
1	6.75	4.5
2	6.75	0.045
3	0.0675	4.5 x 10 ⁻⁴
4		

By inputting each value of P_{loss} obtained in (3) for x , you can calculate the values of $f(x)$ (power loss rate).

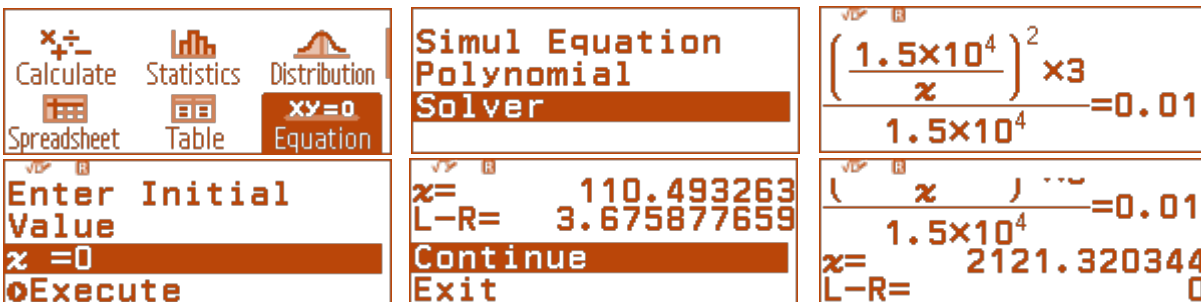
(6) Under the same conditions as (3), find the transmission voltage required to make the power loss rate in the wire 1%.

$$\frac{P_{loss}}{P} = 0.01$$

$$\frac{I^2 R}{P} = 0.01$$

$$\left(\frac{P}{V}\right)^2 R = 0.01$$

$$\frac{\left(\frac{1.5 \times 10^4}{V}\right)^2 \times 3.0}{1.5 \times 10^4} = 0.01 \quad \therefore V \approx 2121 \text{ V}$$



Simul Equation Polynomial Solver

$$\frac{\left(\frac{1.5 \times 10^4}{x}\right)^2 \times 3}{1.5 \times 10^4} = 0.01$$

Enter Initial Value
x = 0
Execute

x = 110.493263
L-R = 3.675877659
Continue
Exit

$$\frac{\left(\frac{1.5 \times 10^4}{x}\right)^2 \times 3}{1.5 \times 10^4} = 0.01$$

x = 2121.320344
L-R = 0

In this example, if the initial value $x = 0$ is used in “Solver”, the solution may not be found in one step. Pressing “Continue” will allow “Solver” to find the correct solution.

The value of x obtained by “Solver” is stored in a variable, so you can calculate the left-hand side as shown below to check your result.



$$\frac{\left(\frac{1.5 \times 10^4}{x}\right)^2 \times 3}{1.5 \times 10^4} = 0.01$$

Atomic Physics

Comparing Nuclear Energy to Everyday Energy Use	Level 1	1
Half-life and Radiocarbon dating	Level 2	2
Simulating Radioactive Decay with Dice	Level 3.....	3

*Level of difficulty

Level 1: Easy

Level 2: Medium

Level 3: Difficult



: Experimental teaching materials



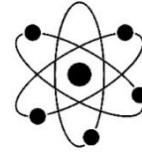
Comparing Nuclear Energy to Everyday Energy Use

Level 1

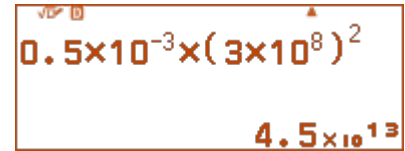
In a nuclear fission reaction, a mass defect of 0.50 g occurs.

(1) Calculate the energy E [J] released by this mass defect.

Use the speed of light $c = 3.0 \times 10^8$ m/s.



$$E = mc^2 = 0.50 \times 10^{-3} \text{ kg} \times (3.0 \times 10^8 \text{ m/s})^2 = 4.5 \times 10^{13} \text{ J}$$



*If you register this value as follows, the following calculations will be convenient.



(2) If one full charge of a smartphone is 10 Wh, how many times can you charge it with the energy from (1)?

$$*1 \text{ Wh} = 3.6 \times 10^3 \text{ J}$$



$$10 \text{ Wh} = 10 \times (3.6 \times 10^3) \text{ J} = 3.6 \times 10^4 \text{ J}$$

Number of charges

$$\frac{4.5 \times 10^{13} \text{ J}}{3.6 \times 10^4 \text{ J}} = 1.25 \times 10^9$$



(3) If the average household electricity consumption is 200 kWh per month, how many households can be powered for one month with the energy from (1)?

$$200 \text{ kWh} = 200 \times 10^3 \times 3.6 \times 10^3 = 7.2 \times 10^8 \text{ J}$$

Number of households

$$\frac{4.5 \times 10^{13} \text{ J}}{7.2 \times 10^8 \text{ J}} = 6.25 \times 10^4$$



(4) How many hours can a 60 W incandescent lamp be lit with the energy from (1)?

$$*1 \text{ W} = 1 \text{ J/s}$$

$$60 \text{ W} = 60 \text{ J/s}$$



$$\frac{4.5 \times 10^{13} \text{ J}}{60 \text{ J/s}} = 7.5 \times 10^{11} \text{ s}$$

$$\frac{7.5 \times 10^{11} \text{ s}}{3600 \text{ s}} \approx 2.1 \times 10^8 \text{ h}$$



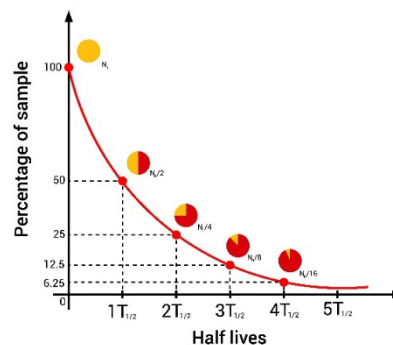


Half-life and Radiocarbon dating

Level 2

The nuclei of the atoms of some elements sometimes emit radiation and transmute into nuclei of atoms of different elements. The time it takes for the number of nuclei of the original element to decrease to half is called the half-life. Letting the number of nuclei of the original element be N_0 , the number of nuclei remaining after t years be N , and the half-life of the element be T years, the following relationship holds.

$$N = N_0 \times \left(\frac{1}{2}\right)^{\frac{t}{T}}$$



(1) For the element ${}^{239}_{94}\text{Pu}$ (plutonium), which has a half-life of 24,110 years, what percentage of the original material remains after 100 years have elapsed?

Since $N = N_0 \times \left(\frac{1}{2}\right)^{\frac{t}{T}}$, $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{100}{24,110}} \approx 0.997 \therefore 99.7\%$



(2) For the radioactive element ${}^{14}_6\text{C}$ (carbon), which has a half-life of 5,730 years, after how many years does $\frac{1}{16}$ of the original material remain?

Since $N = N_0 \times \left(\frac{1}{2}\right)^{\frac{t}{T}}$,

$$\frac{1}{16}N_0 = N_0 \times \left(\frac{1}{2}\right)^{\frac{t}{5,730}}$$

$$\frac{1}{16} = \left(\frac{1}{2}\right)^{\frac{t}{5,730}}$$

$$\left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^{\frac{t}{5,730}}$$

$$\frac{t}{5,730} = 4$$

$\therefore t = 4 \times 5730 = 22920$ years



(3) Measuring the amount of ${}^{14}_6\text{C}$ contained in a wooden artifact discovered in ancient Egyptian ruins, it was found to be $\frac{11}{12}$ that of the amount (abundance) of ${}^{14}_6\text{C}$ in the atmosphere. Assess whether this artifact is from ancient Egyptian times. *The abundance of ${}^{14}_6\text{C}$ in the atmosphere is steady across time.

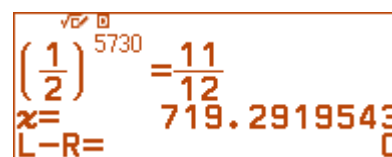


Since $N = N_0 \times \left(\frac{1}{2}\right)^{\frac{t}{T}}$,

$$\frac{11}{12}N_0 = N_0 \times \left(\frac{1}{2}\right)^{\frac{t}{5,730}}$$

$$\left(\frac{1}{2}\right)^{\frac{t}{5,730}} = \frac{11}{12}$$

$\therefore t = 719$ years



\therefore This artifact appears to be much more recent and is therefore unlikely to be from ancient Egypt.



Simulating Radioactive Decay with Dice

Level 3

Treat 200 dice as radioactive isotopes. Assume that any die showing a 1 undergoes radioactive decay. Use the following model experiment to verify how the original number of radioactive isotopes decreases.



Experiment

- Operation 1 (1st attempt)
Roll 200 dice (N_0). Remove the dice that show 1, and record the remaining number of dice as N_1 .
- Operation 2 (2nd attempt)
Roll the N_1 dice. Remove the dice showing 1, and record the remaining number as N_2 .
- Operation k (k^{th} attempt)
Repeat the operation similarly, rolling N_{k-1} dice each time. Stop when the number of remaining dice (N_k) is 25 or fewer.

(1) Complete the following table with the experimental results.

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of 1s	0	34	29	24	19	14	13	12	10	8	7	7
N_k	200	166	137	113	94	80	67	55	45	37	30	23

Rolling N_k dice can be replaced by performing N_k attempts using the [Dice Roll] function on a scientific calculator.



Operation 1

Dice :1	Result Type	Sum	Freq	Rel Fr	Attempts
Attempts :200	List	1	34	0.17	200
Same Result :Off	Relative Freq	2	38	0.19	
Execute		3	26	0.13	
		4	31	0.155	0.17

Number of 1s: 34
 $N_1 = 200 - 34 = 166$

Operation 2

Dice :1	Result Type	Sum	Freq	Rel Fr	Attempts
Attempts :166	List	1	29	0.174	166
Same Result :Off	Relative Freq	2	34	0.2048	
Execute		3	19	0.1144	
		4	25	0.1506	0.1746987952

Number of 1s: 29
 $N_2 = 166 - 29 = 137$

Operation 3

Dice :1	Result Type	Sum	Freq	Rel Fr	Attempts
Attempts :137	List	1	24	0.1751	137
Same Result :Off	Relative Freq	2	26	0.1897	
Execute		3	16	0.1167	
		4	23	0.1678	0.1751824818

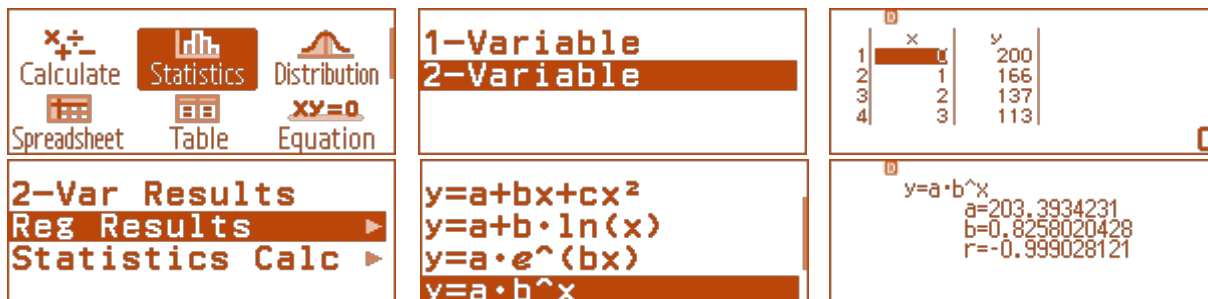
Number of 1s: 24
 $N_3 = 137 - 24 = 113$

Repeat this operation until the number of remaining dice is 25 or fewer.

*Note: In the above example, "Same Result: Off" is selected on the Dice Roll input screen, but data from #1 is actually being used.

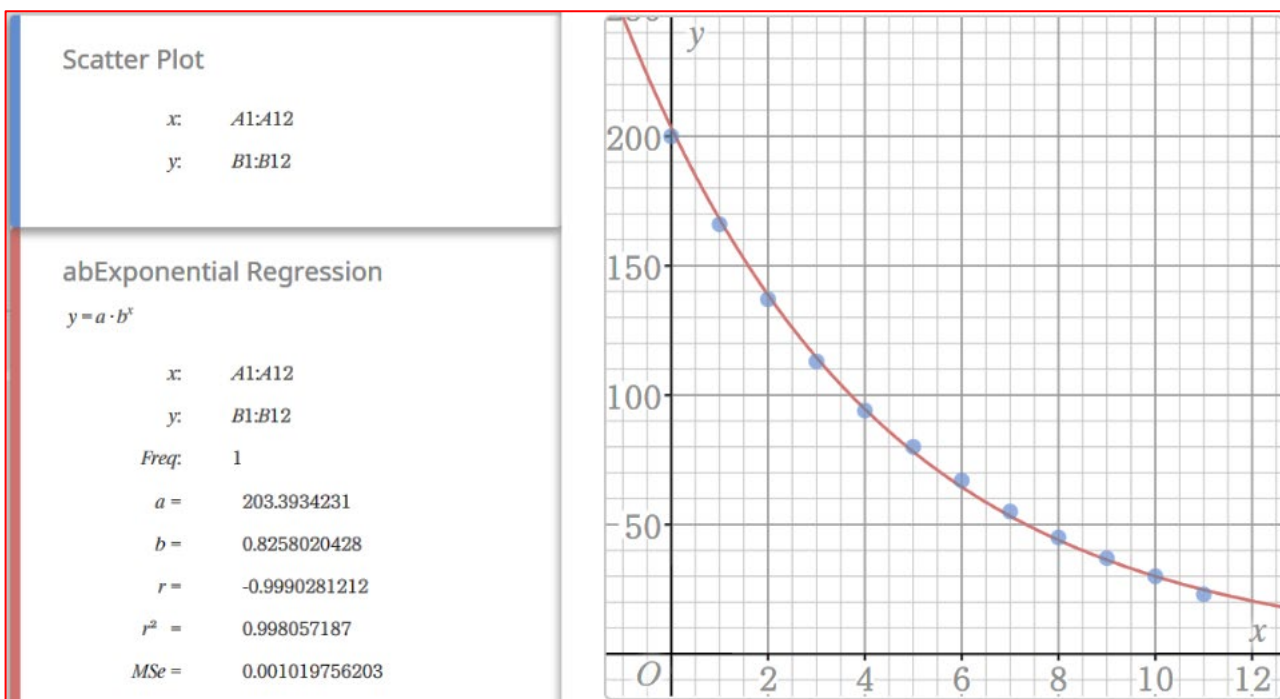
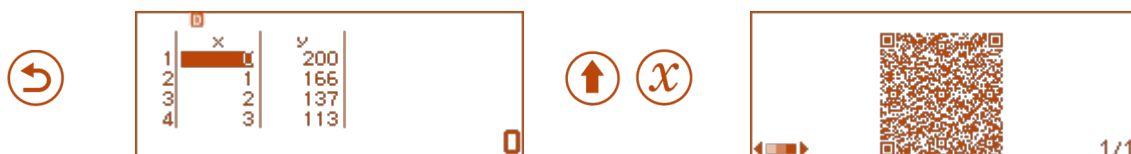
(2) Using the table from (1), plot k on the horizontal axis and N_k on the vertical axis to make a scatter plot. Then find the regression equation.

In Statistics > 2-Variable, input the values of k in the x column and the values of N_k in the y column.



In this experiment, the probability of getting a 1 is $\frac{1}{6}$, and the probability of getting any other number is $\frac{5}{6}$. Therefore, after the operation k , the number of dice remaining is expected to be $N_k = 200 \times \left(\frac{5}{6}\right)^k$. Thus, we use the equation $y = a \cdot b^x$ as the regression model.

To plot the scatter diagram and regression equation, return to the data input screen and generate a QR code.



\therefore The regression equation is $N_k \approx 203 \cdot (0.83)^k$

(3) Using the regression equation, find the approximate number of operations needed for the number of isotopes to decrease by half (100 isotopes). This value is called the half-life T .

Calculations using the regression equation can be performed from [Statistics Calc].

<table border="1"> <tr><th>x</th><th>y</th></tr> <tr><td>1</td><td>200</td></tr> <tr><td>2</td><td>166</td></tr> <tr><td>3</td><td>137</td></tr> <tr><td>4</td><td>113</td></tr> </table>	x	y	1	200	2	166	3	137	4	113	2-Var Results Reg Results ▶ Statistics Calc ▶	$y=a+bx+cx^2$ $y=a+b \cdot \ln(x)$ $y=a \cdot e^{(bx)}$ $y=a \cdot b^x$
x	y											
1	200											
2	166											
3	137											
4	113											

Enter the value of y (100), and then use the \hat{x} function to estimate the value of x .

100	Statistics ▶ Func Analysis ▶ Probability ▶ Numeric Calc ▶	Summation ▶ Mean/Var/Dev... ▶ Min/Max ▶ Regression ▶				
<table border="1"> <tr><th>a</th><th>b</th></tr> <tr><td></td><td>\hat{x}</td></tr> </table>	a	b		\hat{x}	$100 \hat{x}$ 3.709358678	
a	b					
	\hat{x}					

∴ The half-life T is 3.7

Furthermore, this value can be confirmed to match the value read from the graph as follows.



Substitute the half-life T and $N_0 = 200$ obtained in (4)(3) into the half-life formula: $N_k = N_0 \cdot \left(\frac{1}{2}\right)^{\frac{k}{T}}$.

Verify whether the result matches the experimental data when $k = 0, 1, 2 \dots$.

$$N_k = N_0 \left(\frac{1}{2}\right)^{\frac{k}{T}} = 200 \cdot \left(\frac{1}{2}\right)^{\frac{k}{3.7}} \quad \text{Verify this in the table.}$$

Calculate Statistics Distribution Spreadsheet Table Equation	$f(x) = 200 \times \left(\frac{1}{2}\right)^{\frac{x}{3.7}}$	<table border="1"> <tr><th>x</th><th>f(x)</th></tr> <tr><td>0</td><td>200</td></tr> <tr><td>1</td><td>165.83</td></tr> <tr><td>2</td><td>137.5</td></tr> <tr><td>3</td><td>114.01</td></tr> </table>	x	f(x)	0	200	1	165.83	2	137.5	3	114.01
x	f(x)											
0	200											
1	165.83											
2	137.5											
3	114.01											

CASIO Teaching Materials
Physics Materials with fx-991CW

Publisher: CASIO COMPUTER CO., Ltd.

Date of Publication: 2026/3/31 (1st edition)

<https://edu.casio.com>

Copyright © 2026 CASIO COMPUTER CO., LTD.

(1) These teaching materials may be freely reproduced and distributed by teachers for educational purposes.

* However, publication on the Web or simultaneous transmission to an unspecified number of people is prohibited.

(2) Any reproduction, distribution, editing, or use for purposes other than those listed above requires the permission of CASIO COMPUTER CO., LTD.

(3) QR Code is a registered trademark of DENSO WAVE INCORPORATED in Japan and in other countries.



**Boost
Your
Curiosity**